

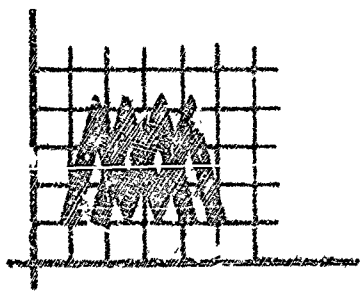
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Study of Dynamic Response of Stranded Wire Springs.

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FINAL TECHNICAL REPORT

to

Headquarters
U.S. Army Weapons Command
Rock Island, IL 61201
ATTN : AMSVE-RET

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Cheshire, December 1970.

Study of Dynamic Response in Stranded Wire Springs.

Final Technical Report

on

Contract DAAF03-70-C-0078

to

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Study of Dynamic Response of Stranded Wire Springs.

A B S T R A C T

A total of 60 TD-recordings were made of 5 different test springs, 42 records in impact loading at velocities up to 560 inch/sec, and 18 records on the sudden release of spring, at extension velocities up to 830 inch/sec.

Each record was evaluated with respect to surge time (both wave head and wave peak), velocities, minimum height reached, etc..

For single wire springs, the formulas of the Surge Wave Theory were recapitulated and summarized, ^{and} ~~in Section C. In Section D, corresponding~~ formulas were derived for stranded wire springs. These formulas use lumped values as parameters, such as the active weight of spring (or per coil) and the measured rate of the spring (or coil), instead of wire diameter d and mean coil diameter D . ()

When using the measured initial rate of spring in these formulas, good agreement was reached between the calculated and the measured values. Therefore, a set of formulas is available now for stranded wire springs which predict dynamic values such as surge time, surge velocity, extension velocity in sudden release and induced load and stress within a few percent of the measured values.

Stranded wire springs were found to show a few phenomena over those observed on single wire springs: The surge velocity is not a constant for all compression levels of spring, but increases slightly with increasing compression. The slope of the wave head has a pronounced tendency to broaden during propagation, so that head and peak of wave have somewhat different surge velocities (about 10% difference). The wave head disappears in the records after from 2 to 3.5 surge times T , as compared to $6T$ or $8T$ in single wire springs.

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Study of Dynamic Response of Stranded Wire Springs.

A. Scope of Work.

1. Scope of work in general.

Photograph the transient wave propagation in stranded wire springs with a time-displacement camera under prescribed conditions of impact velocity, mass weight and spring preloads.

Evaluate time-displacement records and obtain measured values for time, coil displacement, velocity, acceleration and spring force. Determine the transient wave velocity, the dynamic spring force and stress from the TD-records. Some tests will also be performed on pre-compressed stranded wire springs released suddenly to determine expansion velocity and characteristics.

Conduct a theoretical analysis and establish formulae for the calculation of the dynamic load and stress and the transient wave velocity in stranded wire springs.

Correlate theoretical results with experimental data and prepare report on findings.

2. Number of test springs.

Five stranded wire test springs, two of the 3-strand type and three of the 7-strand type are to be investigated:

3-strand drive spring for SPIW Rifle;

7-strand drive spring for SPIW Rifle;

3-strand drive spring for Cal..50 M85 Machine Gun;

7-strand drive spring for Cal..50 M85 Machine Gun;

7-strand drive spring for 20 mm Machine Gun.

3. Number of TD-records to be taken.

The Contractor shall record a total of 50 time-displacement records, with usually 2 records each for a given condition of spring loading. These 24 pairs of records may be composed as follows:

5 types of stranded wire springs under impact,
with two levels of preload each :

10 conditions

2 types of stranded wire springs under sudden
release, with 2 levels of preload each :

4 conditions.

This makes for 14 different conditions or 28 records total. Another 22 records are extras, to be taken at different film speeds and magnifications, as the need arises during the program.

4. Evaluation of TD-records.

All the records will be evaluated regarding measured values of time, coil displacement, velocity, acceleration, surge velocity, spring force and stress. In addition, the damping of the surge waves over time and displacement will be evaluated. Also, it will be endeavored to measure the degree of damping of a particular spring by a numerical value, say by an exponent a in the energy term e^{-at} , supposed the energy decay follows such an exponential law.

5. Theoretical analysis of wave propagation in stranded wire springs.

The transient wave velocity will be derived from the General Wave Equation:

$$d^2y/dt^2 = c^2 \cdot d^2y/dx^2 \quad (1)$$

This general equation must be modified to account for the energy decay due to damping as inherent in stranded wire springs. At least an approximate solution will be sought for determining the influence of damping on the wave velocity.

6. Technical Reports.

The results of the recordings and the analysis will be adequately covered in the form of Technical Reports, 3 Quarterly Reports and one Final Report. The Third Quarterly Report may be included in the Final Report, if the Scope of Work has been completed by that time. These reports will contain some of the TD-records taken as examples, which are discussed in detail. Of the prints not contained in these reports, an extra set of prints will be forwarded separately.

B. Compiling basic spring data.

1. General.

On each of the 5 test springs, the basic dimensions were measured, before as well as after the testing. Some of the springs (SPIW-7 and M85-7) took a definite set during the testing, while the other 3 springs maintained their free height.

The average pitch of the middle coils of the springs, h_p , at free height was also measured, over a length comprising from 30 to 60 coils. With h_p known, the displacement magnification q of a TD-record can be determined more accurately than from the free height H_p of spring, where the actual ends of the spring cannot be marked by special traces.

The force-deflection rate P/F of the springs was also calculated, by means of the conventional formula for single wire springs and simply adding the rates of the 3 or 7 single wire springs. Also, measured force-deflection diagrams $P(F)$ were made available for each of the 5 test springs by AMSWE-RET. It was felt that a comparison between the calculated and measured spring rates (static rates) is a necessary part of this study.

2. SPIW drive spring, 3 strand.

The basic data are compiled in data sheet X1, in Appendix X. Its measured force-deflection diagram $P(F)$ is plotted in Diagram 1, Appendix X. This spring maintained its free height during the testing. The calculated rate $P/F = 1.38$ lbs/inch and the measured rate over the first 5" of stroke (1.40 lbs/inch) are in close agreement.

3. SPIW drive spring, 7 strand.

Here an inner guide rod of .266" was selected. The spring ends were found to have a somewhat lesser diameter than the middle coils, binding on a somewhat larger rod of .272" diameter.

Data sheet X2 in Appendix X gives the basic data and Diagram 1, page X3, illustrates the measured P/F -diagram. Here the rate $P/F = 1.85$ lbs/inch calculated under simplifying assumptions is substantially less than the rate $P/F = 2.10$ lbs/inch measured over the first 5" of stroke. This spring set from $H_p = 14.26"$ to 13.80" during testing.

4. 3-strand M85 drive spring.

The basic data are compiled on page X4 of Appendix X. This spring is rather husky, with an index D/d of only 8.62. Furthermore, it is coiled extremely steep, with an uncorrected solid stress S_s approaching 190,000 psi.

Diagram 2 on page X6 of Appendix X illustrates the measured force-deflection diagram, $P(F)$, according to the data furnished by AMSWE-RET. Over the first 5 inches of stroke, where the characteristics is rather linear, a spring rate $R = 11.20$ lbs/inch was measured. The spring rate calculated from the spring data [assuming 3 wires in parallel] is $R = 10.45$ lbs/inch, i.e., about 7 % less.

The spring experienced only a minor set during testing, from $H_f = 18.30"$ to $H_f = 18.20"$.

5. 7-strand M85 drive spring.

The basic data are compiled on page X5 of Appendix X and the measured force-deflection diagram $P(F)$ is illustrated in Diagram 2, page X6, of the Appendix X. The calculated rate [parallel single wire springs assumed] of $R = 3.29$ lbs/inch was found to be substantially less than $R = 4.0$ lbs/inch as measured over the first 3 inches of stroke. Notice the great difference in rate and characteristics with respect to the 3-wire strand spring for the same gun.

This spring was found to take a substantial set during the testing, from $H_f = 18.26"$ to $H_f = 16.90"$. This large set is mostly due to the omission of the buffer in a few tests, where the spring was then crushed solid by excessive kinetic energy of the impact mass.

6. 7-strand 20 mm gun drive spring.

The basic data are compiled on page X7 of the Appendix X and the measured force-deflection characteristics $P(F)$ is illustrated in Diagram 3, page X8, of Appendix X. The calculated rate [parallel wires] of $R = 6.25$ lbs/inch is well below the measured rate of $R = 7.50$ lbs/inch (over the first 10 inches of stroke).

This spring took only a very minor set during the testing, from $H_f = 24.65"$ to $H_f = 24.5"$ and was found to have recovered to the initial free height after weeks.

C. Summary of surge wave formulas on single wire springs.

1. Terminology.

a. Dimensions, in [inch].

d	Wire diameter
d*	Diameter of wire strand
D _o	Outer diameter of coils
D _i	Inner diameter of coils
D	Mean coil diameter
H	Height or length of spring
H _f	Free height
H _s	Solid height
H ₁	Initial height (preloaded) in impact loading
H ₂	Initial height in sudden release recordings
h	Pitch of coil
F	Deflection of spring
f	Deflection per coil
x	Wire coordinate, measured along the wire
L ₁	Length of wire of one coil
L	Length of wire of entire spring (active coils)
y	Deflection of a point of wire of coordinate x

b. Times, in [sec] or [ms].

t	Time in general (variable)
t = 0	Instant of impact or sudden release
T ₁	Wave propagation time or "surge time", per coil
T	Wave propagation time or "surge time" through spring (single length)
T _h	Surge time of the wavehead in first transition
T _s	Separation time of rear spring end in sudden release
4T	Period of free vibration of spring in sudden release, when rear spring end is fixed.

C1. Terminology, continued.

c. Velocities, [inch/sec].

- c Velocity of propagation of wave along the wire, also called "surge velocity".
- v_0 Impact velocity of impact mass on spring, also extension velocity of spring end in sudden release
- $v(x,t)$ Velocity of any point on wire axis, of wire abscissa x , at the instant of time t .
- Δv Kinetic amplitude of surge wave

d. Weights and forces, in [lbs].

- $w = m \cdot g$ Weight of spring (active coils only)
- $W = M \cdot g$ Weight of impact mass
- P Load at any spring point, also force at spring end
- P_1 Preload of spring in impact loading
- P_2 Preload of spring in sudden release recording
- P_s Spring force at solid height
- $P(F)$ Static force-deflection diagram
- ΔP Change of force induced by an impact of velocity v (Force amplitude of surge wave)

e. Stresses, in [psi] .

- G Modulus of elasticity in torsion
- S Stress at any point of spring (at wire surface), (shear).
- S_1 Initial (static) stress of preloaded spring (impact loading)
- S_s Static stress at solid height of spring
- ΔS Change of stress induced by impact of velocity v (stress amplitude of surge wave)

C1. Terminology, continued.f. Other values

Dimension

g	Acceleration due to gravity, 386.4	[inch/sec ²]
γ	Weight of spring material per inch ³	[lbs/in. ³]
γ_s	.283 lbs/inch ³ for spring steel	
$P = P/F$	Force-deflection rate of spring	[lbs/inch]
$R_1 = P/f$	Force-deflection rate per coil	[lbs/in./coil]
$C = D/d$	Spring index	[---]
N_t	Number of total coils in spring	[---]
N_2	Number of active coils only	[---]
u	Potential at a point of spring, by definition	[inch/sec]
Δu	Change of potential induced by impact velocity v (potential amplitude of surge wave as opposed to kinetic amplitude v)	
a	Radius of spiral of single wire around strand centerline	
b	Length of lay of single wire, along strand centerline	
b^*	Length of lay of single wire, absolute	—

8

2. Basic wave equation and surge velocity.

In the basic wave equation

$$\boxed{d^2y/dt^2 = c^2 \cdot d^2y/dx^2} \quad (1)$$

the constant c denotes the velocity of propagation or "surge velocity" in the linear medium. For a helical compression spring, this surge velocity c is given by the formula

$$\boxed{c^2 = \frac{R_{pf}}{\gamma/g} \cdot \frac{L_1}{(A + 4I_p/D^2)}} \quad [\text{inch/sec}] \quad (2)$$

where

$R_{pf} = R_1$: force-deflection rate of one coil , [lbs/inch/coil]

A : Cross-sectional area of wire , [inch²]

I_p : Polar moment of inertia of wire cross-section, [inch⁴].

For a round wire spring, these values are:

$$R_{pf} = R_1 = \frac{G}{8} \cdot \frac{d^4}{D^3} \quad (3)$$

$$A = (\pi/4) \cdot d^2 \quad (4)$$

$$I_p = (\pi/32) \cdot d^4 \quad (5)$$

The length of one coil, strictly speaking, is slightly longer than $\pi \cdot D$, by a factor k slightly above 1:

$$L_1 = k \cdot \pi \cdot D \quad (6)$$

For a roundwire spring, the formula (2) for c then becomes:

$$\boxed{c^2 = \frac{d^2}{D^2} \cdot \frac{G \cdot g}{2\gamma} \cdot \frac{k}{1 + d^2/2D^2}} \quad (7)$$

With $k = 1.01$ (conventional choice) and spring index $D/d = 7$ (mean value), the third factor in equation (7) becomes 1 and we have the simplified formula:

$$\boxed{c = \frac{d}{D} \cdot \sqrt{\frac{G \cdot g}{2\gamma}}} \quad (8)$$

For a spring from steel wire, this formula further simplifies to:

$$\boxed{c = 88,560 \cdot (d/D)} \quad [\text{inch/sec}] \quad (9)$$

3. Surge time and natural frequency.

We distinguish between the surge time T through the entire spring and the surge time T_1 per coil. In evaluation of TD-records, the latter should be given priority, due to the slight uncertainty regarding the accurate number of active coils in a spring:

$$T_1 = T/N = L_1/c = k \cdot \pi \cdot D/c \quad [\text{sec/coil}]. \quad (10)$$

Assuming $k = 1.01$ as is conventional, it follows from (8) and (10):

$$T_1 = T/N = \frac{3.17 \cdot D^2}{d} \cdot \sqrt{\frac{2 \cdot \chi}{g \cdot G}} \quad (11)$$

With mean values of spring steel:

$$T_1 = T/N = 35.8 \cdot 10^{-6} \cdot D^2/d \quad (12)$$

When defining the active solid height

$$H_s = N \cdot d, \quad (13),$$

an alternate formula for the surge time is obtained:

$$T = 35.8 \cdot 10^{-6} \cdot (D/d)^2 \cdot H_s \quad (14)$$

The latter form shows clearly, that the surge time T increases with the square of the spring index D/d .

Natural frequency:

If a spring is fastened with one end, the spring will oscillate at a fundamental natural frequency, where the period of vibration comprises 4 surge times T :

$$N_1 = 1/4T \quad [\text{sec}^{-1}] \quad (15)$$

With mean values of spring steel:

$$n_1 = 7000 \cdot d/(D^2 \cdot N) \quad (16)$$

With both spring ends fastened, the period of the fundamental free vibration comprises only 2 surge times T and the fundamental natural frequency n_1 becomes twice the above value of equation (16).

4. Single surge wave and amplitudes.

The forced motion $v(t)$ of a spring end or of an intermediate point x of spring generates a "single surge wave", which has a kinetic amplitude of same velocity v and a potential amplitude, which can be expressed in 4 different ways:

As a change of load or force, ΔP , in [lbs] ;
 as a change of stress (uncorrected), ΔS , in [psi] ;
 as a change of deflection per coil, Δf , in [inch/coil];
 as a change of potential, Δu , in [inch/sec].
 By definition, $u = v$.

The following formulas have been derived:

$$\Delta P/v = R_1 \cdot T_1 = (3.17/8) \cdot (d^3/D) \cdot \sqrt{2 \gamma \cdot G/g} \quad (17)$$

$$\Delta S/v = (\Delta P/v) \cdot (\Delta S/\Delta P) = \sqrt{2 \gamma \cdot G/g} \quad (18)$$

$$\Delta f/v = T_1 = 3.17 \cdot (D^2/d) \cdot \sqrt{(2 \gamma / g \cdot G)} \quad (19)$$

$$\Delta u/v = 1 \quad (\text{by definition}) \quad (20)$$

With mean values for spring steel, these formulas specialize to:

$$\Delta P/v = 51.5 \cdot d^3/D \quad [\text{lbs/inch/sec}] \quad (21)$$

$$\Delta S/v = 131 \quad [\text{psi/inch/sec}] \quad (22)$$

$$\Delta f/v = 35.8 \cdot 10^{-6} \cdot D^2/d \quad [\text{inch/coil per inch/sec}] \quad (23)$$

Here the surprising result is: The stress change induced in the spring by a forced velocity v (such as impact) is proportional to the velocity v , but independent from the spring dimensions d and D . Each inch/sec of forced velocity changes the stress of the spring point in question by 131 psi.

D. Modification of formulas for stranded wire springs.

1. Derivation of general formula for surge velocity.

We return to the basic wave equation (1) and to the general formula for the surge velocity, which is also valid for stranded wire springs:

$$c^2 = \frac{R_1 \cdot L_1}{(\gamma/g) \cdot (A + 4 \cdot I_p/D^2)} \quad (2)$$

where $R_1 = R_{DF}$: force-deflection rate of one coil, in [lbs/inch],
and L_1 : length of wire in one coil, in [inch] .

With the exception of the unusually husky 3-strand M85 drive spring ($D/d = 8.62$), the spring index D/d of the other 4 test springs ranges from 11.3 to 15. With an index of 14, the term $4 \cdot I_p/D^2$ drops to only 0.25% of the term A and therefore can be rightfully neglected. When further introducing the weight of one coil:

$$W_1 = \gamma \cdot A \cdot L_1 \quad [\text{lbs/coil}] ,$$

equation (2) becomes:

$$c^2 = L_1^2 \cdot (R_1 \cdot g/W_1) \quad (24)$$

For stranded wire springs, this more general formula for the surge velocity should be used, for the following reasons:

- a) The length L_1 of one coil can be substantially longer than the conventional value of $3.17 \cdot D$ [i.e., $k = 1.01$ in formula (6)] for single wire springs.
- b) A stranded wire spring has a non-linear force-deflection characteristics, as the measured diagrams in Appendix X clearly show. The spring rate R of the entire spring and therefore also the spring rate R_1 of one coil increase with increasing deflection. This is due to the fact, that the wires are not only stressed in pure torsion as with a single wire spring, but have a more complex stress distribution pattern including tension, bending and also friction. There is no satisfactory and accurate method to derive the progressive force-deflection characteristics of a stranded wire spring to obtain a reliable formula. But measured values of the rates R and R_1 could be used instead.

For these reasons, it is preferred to leave formula (24) in its general form, with L_1 and R_1 as parameters.

2. Derivation of formulas for surge times.

From equation (10), the surge time per coil, T_1 , and of the entire spring follow to be:

$$T_1 = L_1/c \quad [\text{sec}] \quad , \quad (10a)$$

and $T = N \cdot T_1 = N \cdot L_1/c = L/c \quad [\text{sec}] \quad (10b)$

When introducing c from formula (24), it follows:

$$T_1 = \sqrt{W_1/(R_1 \cdot g)} \quad [\text{sec}] \quad , \quad (25)$$

and $T = \sqrt{W/R \cdot g} \quad [\text{sec}] \quad . \quad (26)$

Here W denotes the active weight of the entire spring and may be calculated from the formula:

$$W = N \cdot W_1 = 0.785 \cdot d^2 \cdot L_1 \cdot [\text{lbs}] \quad . \quad (27)$$

Here the wire length per coil, L_1 , has to be calculated and for the rate $R_1 = N \cdot R$, the measured value of R may be used.

The surge times T_1 and T are proportional to the square root of the weight per coil, W_1 , and inversely proportional to the spring rate per coil, R_1 . The surge time T , of course, is also proportionate to the number of active coils, N .

Natural frequency:

For a spring with one fixed end and one free end, the period of free vibration comprises again 4 surge times and the fundamental natural frequency becomes:

$$n_1 = 1/4 \cdot T \quad [\text{sec}^{-1}] \quad . \quad (15)$$

And for a spring with both ends fixed, the period of free vibration comprises only 2 surge times and the fundamental natural frequency becomes:

$$n_1 = 1/2T \quad [\text{sec}^{-1}] \quad . \quad (15a)$$

3. Amplitudes of single surge wave in stranded wire springs.

The impact of a mass on a spring end with velocity v (or any forced motion) generates a "single surge wave", which has a kinetic amplitude (coils are forced to move with same velocity v) and a potential amplitude, which can be expressed in various ways:
 As a change of deflection per coil, Δf , in [inch/coil] ;
 as a change of load or force, ΔP , in [lbs] ;
 as a change of stress (uncorrected), ΔS , in [psi] .

Deflection per coil:

During a time interval equal to the surge time per coil, T_1 , the surge waves traverses one coil, while the end of coil makes a displacement ($v.T_1$). Therefore, the deflection per coil becomes:

$$\boxed{\Delta f = T_1 \cdot v} \quad [\text{inch/coil}] . \quad (28)$$

Load induced:

This deflection per coil induces an additional force

$$\boxed{\Delta P = R_1 \cdot \Delta f = R_1 \cdot T_1 \cdot v} \quad [\text{lbs}] . \quad (29)$$

Stress induced:

In a single wire spring, the maximum torsion stress S (uncorrected) has been found to be independent from the dimensions d and D of the spring:

$$\boxed{\Delta S = \sqrt{2 \gamma \cdot G/g} \cdot v} \quad [\text{psi}] , \quad (18)$$

or, for a steel spring: $\Delta S = 131 \cdot v \quad [\text{psi}] . \quad (22)$

In a stranded wire spring, however, the stress distribution pattern is more complex. The impact energy is not only stored in torsion stresses, but also in bending and tension stresses (besides friction). Therefore, the actual maximum torsion stress will be slightly less than the value calculated from above formulas (18) and (22).

4. Calculation of wire length per coil.

The wire length per coil, L_1 , enters into the formula for the surge velocity c and also into the formulas for the surge times, by way of the weight per coil, W_1 . Therefore, L_1 must be calculated as accurately as possible.

The coil length L_1 of a single wire in a stranded wire spring is greater than the projected circumference $3.14.D$, for 2 reasons:

- a) due to the pitch of spring at free height, h_f ;
- b) because the wire (except the core wire) forms a spiral or helix around the centerline of the straight or coiled strand.

Table I on page Y1 of Appendix Y illustrates the procedure of calculating L_1 for the 5 test springs under consideration.

The length of the coil centerline follows from the formula:

$$L_1' = \sqrt{(3.14.D)^2 + h_f^2} \quad [\text{inch}] \quad (30)$$

The ratio $L_1'/3.14.D$ is found to be from 1.018 to 1.034, i.e., a percentage increase of the centerline length from 1.8% to 3.4% over $3.14.D$.

The second step is the calculation of the length of the spiral of a wire with respect to the centerline of the strand:

First, the number of wires wrapped around the centerline over one coil was measured on the test springs, ranging from 7 turns for the M85-3 spring to 17 turns for the 20-mm-7 spring. From this, the length of one lay, designated by b , was calculated. b was found to range from .367" to 1.15" .

Second, the radius a of the spiral was determined (see illustration of radius a on the bottom of Table I), as well as the circumference $2\pi a$. The increase of the wire length L_1 over that of the strand centerline is then described by the ratio:

$$L_1/L_1' = b^*/b = \sqrt{1 + (2\pi a/b)^2} \quad (31)$$

This length increase was found to be from 4.5 to 7.6 %, i.e., about twice as great as the relative length increase from the pitch.

Altogether (pitch and spiral effect combined), the length L_1 of a single wire coil is substantially greater than $3.14.D$, by from 7% (SPIW-3 spring) to 10.8% (20 mm-7 spring) .

5. Numerical calculation of surge times and surge velocity.

The results of these numerical calculations are given in Table II, page Y2, in Appendix Y.

First the cross-sectional area of the individual wires were calculated and added up to the cross-sectional area A of the entire strand. By multiplication with the length of coil L_1 (from Table I), the weight per coil, W_1 , is obtained. The active weight of the spring has also been calculated in Table II.

We now consider the formulas (25) and (26) for the calculation of the surge times T_1 and T . Due to the non-linear force-deflection characteristics (see Diagrams 1 to 3 in Appendix X), it becomes here somewhat doubtful, what values should be used for the spring rates R_1 and R . Since no decision can be made prior to a comparison with the test results, two alternate calculations have been made using the following assumptions:

Assumption A:

Here the spring rate of each single wire in the strand was calculated by means of the well-known formula for single wire springs and these rates were simply added to obtain the rate of the stranded wire spring. These rates were already given in the data sheets of the springs in Appendix X.

Assumption B:

Here the rate of the spring was evaluated from the measured force-deflection diagrams of the springs (Diagrams 1 to 3 in Appendix X). Only the initial, linear portion of the diagram was used, over a stroke anywhere from 3" to 10". These rates were also given in the data sheets of the springs in Appendix X. The measured rates were found to be always higher than the calculated rates, with deviations up to 20%. The deviations were found to be greater with 7-wire strand springs than with the 3-wire strand springs.

Under each assumption, the surge velocity c has also been calculated for the 5 test springs. With assumption B, shorter surge times and higher surge velocities are obtained.

6. Numerical calculation of amplitudes of single surge wave.

In a single surge wave, the potential amplitude is always proportional to the impact velocity v . It can be expressed either as deflection per coil, Δf , induced load, ΔP , induced stress, ΔS .

Deflection per coil, Δf :

According to formula (28), the deflection per coil Δf generated by 1 inch/sec of impact velocity is equal to the surge time per coil, T_1 . Therefore, the T_1 -values given in Table II of Appendix Y already establish the ratio $\Delta f/v$ for each of the 5 test springs. For example (using assumption B), in the SPIW-3 spring, each inch/sec of impact velocity generates a deflection per coil $\Delta f = .000157$ [inch]. With a total deflection per coil $f_s = h_f - d^* = .231 - .073 = .158$ ", it would take an impact velocity $v_0 = 1000$ inch/sec to render the spring coils solid in one impact. *for P. 2*

The reverse holds also true: A precompression by .000157 inch/coil will generate an extension velocity of 1 inch/sec, upon sudden release. If the spring is suddenly released from solid height, an extension velocity $v_0 = 1000$ inch/sec can be expected. This is the limit velocity of the spring.

Induced force, ΔP :

According to formula (29), the ratio $\Delta P/v$ is equal to $(R_1 \cdot T_1)$ and these numerical values are also given in Table II. For example (second assumption), in the SPIW-3 spring, each inch/sec of impact velocity induces an additional force $\Delta P = .0163$ [lb]. Or, in other words, an impact velocity of $1/.0163 = 61.5$ inch/sec is required to raise the force of the spring by 1 lb.

Induced stress, ΔS :

Here it is suggested to use the formula for single wire springs, (18), which takes the very simple form of formula (22) for steel springs. Each inch/sec of impact velocity raises the stress by 131 psi. This relationship is considered a good approximation also for stranded wire springs, where the force-stress relationships are more complex than in a single wire spring.

E. Preparation of test fixtures for recording spring response.

1. Fixture for impact-loading of springs.

On a heavy and rigid bench stand, 2 end supports were mounted at a distance of about 56", to receive the ends of an inner guide rod.

This guide rod consists of two sections:

A righthand section of .812" diameter guiding the accelerating spring, which can be preloaded to various heights to impart any desired velocity to the impact mass.

A lefthand section of rod to guide the test spring. This section was made interchangeable, having diameters of .266", .300", .375" and .750".

Between the accelerating spring and the test spring, the impact mass is arranged. This mass is guided on the outside, in a tube of 1.50" inner diameter. This was made necessary by the rather thin guide rods for some of the test springs, which were much too flexible to guide a heavy impact mass. The guidetube contains a longitudinal window for viewing of the test spring by the time-displacement camera and also longitudinal slots for a cocking handle of the impact mass and for triggering means.

The opening of the camera shutter is synchronized with the release of the impact mass from a sear mechanism.

An impact mass of about 2 lbs had been specified. However, the 5 test springs were found to be widely different in energy capacity and it was found proper to adjust the weight of the impact mass somewhat to the strength of the springs. Therefore, 3 different weights were used in this test program:

<u>SPIW test springs</u> :	W = 1.15	lbs	,	L = 3.15"	;
<u>M85 test springs</u> :	W = 2.15	lbs	,	L = 6.05"	.
<u>20 mm test spring</u> :	W = 3.25	lbs	,	L = 5.25"	.

The specified range of impact velocities

v_0 from 30 to 40 ft/sec

was somewhat amplified to range from

$v_0 = 25$ ft/sec to close to 50 ft/sec.

2. Fixture for sudden release of preloaded springs.

Here the 2 end supports were mounted closer together, at a distance of about 34", receiving the ends of the inner guide rods of the test springs. 3 of the 5 test springs were submitted to sudden release tests: SPIW-3, SPIW-7 and M85-7 drive springs.

A sear mechanism had to be developed for the sudden release of the precompressed springs. A sear blade holds one side of the end coil of spring, also touching the guide rod. Then this sear blade is suddenly withdrawn by the core of a solenoid, in synchronisation with the opening of the camera shutter.

The test spring was always released alone, with no adjacent mass to be driven. Two levels of preload were used, one close to solid compression and the other close to half compression. Here the rear end of spring was only supported in the rearward direction, so that it could fly off forward at the end of the spring expansion.

In another arrangement, the rear spring end was fastened to the inner guide rod, permitting no freedom in either direction. In this case, the spring performs a free vibration, in periodic fashion.

P. 3-strand SPIW drive spring:

Discussion of TD-records on impact-loading.

1. Review of TD-records delivered.

A total of 8 TD-records was taken and delivered: Two each for each of the 4 combinations of high or low impact velocity v_0 and no preload (spring at free height H_p) and a substantial preload. 4 of these TD-records are contained in this report, while a second set of 4 TD-records taken under the same operating conditions is being delivered separately.

Operating condition		No. of record in report	No. of record in second set
Impact velocity	Precompression		
high	none	70-723 .	722
high	yes	725	726
low	none	728	727
low	yes	730	731

All records on photo-recording paper.

Note : A block of 20 consecutive numbers has been reserved for the TD-records taken on the 3-strand SPIW drive spring.

On impact-loading : TDR 70-721 to 70-732.

on sudden release : TDR 70-733 to 70-740 (See Section G).

2. Discussion of TD-record 70-723. (high, no preload)

Operating conditions:

The timing marks given at intervals of 1 millisecc indicate a film speed $p = 201$ inch/sec, with the time axis running from right to left. The magnification

$$q = .387x$$

has been determined from the distance of the horizontal coil traces prior to impact, when the coils were at rest and at free height.

A rubber buffer has been installed in parallel to the test spring, to be contacted at a height of test spring $H = 7"$.

The impact velocity of the impact mass of 1.15 lbs was evaluated to be

$$v_0 = 478 \text{ inch/sec.}$$

Response of spring:

The test spring decelerates the motion of the impact mass only to a minor extent, with the additional buffer doing most of the stopping. The minimum height $H_{\min} = 5.50"$ observed comes very close to the solid height H_s of the spring. The impact mass leaves the spring again at a rebound velocity $v_o' = 294$ inch/sec.

The impact of the mass generates a step wave, which traverses the spring at evidently constant surge velocity. However, while the acceleration of the front coils is rather sharp and almost instant-like, the acceleration of the later coils comprises a longer and longer time interval, as the wave progresses. For this reason, it becomes difficult to determine the surge time and surge time per coil accurately. In reading the surge time per coil,

$$T_1 = .160 \text{ millisecond/coil} ,$$

the very first motion response of the previously resting coils has been considered. The surge time T , however, was read in such a way, that the wave had reached a spring end with its full amplitude rather than with its faint head only. It was found:

$$T = 11.85 \text{ milliseconds.}$$

Up to the time $t = 2T$, with only 2 waves superposing, the imposed deflections per coil are absorbed without clashing between coils. After $t = 2T$, however, due to the superposition of 3 wave amplitudes, the front coils start to clash and this clashing is propagated toward the rear coils in a much shorter time interval than T . Notice the almost vertical wave front of the wavehead after 3 superpositions.

After about 3 transitions of the wave, in spite of the high impact velocity and steep wavehead, the wavehead cannot be discerned anymore. The coils now share rather equally in the overall deflection imposed on the spring, which is a most desirable feature. This quick equalization of the deflection over all the coils is much more pronounced than in a single wire spring and is evidently caused by high inherent damping.

3. Discussion of T₂-record 70-725. (high with preload)

Operating conditions:

Here the test spring was precompressed, to a height $H_1 = 13.0"$. The pitch h_1 of the precompressed coils had been measured prior to recording and the magnification $q = .387x$ was confirmed by this measurement. The impact velocity of the mass is found from the record:

$$v_0 = 465 \text{ inch/sec.}$$

Response of spring:

Again, the test spring fails to decelerate the impact mass substantially and reaches again a minimum height of 5.5", close to solid height. [The front coils are somewhat hidden by the precompressing means]. The surge time T to the rear end of spring is measured to be

$$T = 12.0 \text{ ms}$$

and the surge time per coil is found to be $T_1 = .155 \text{ ms/coil}$. After $t = T$, with the start of 2 wave amplitudes being superposed to the precompression, the rear coils clash and the reflection of the wave to the front end of spring takes only about half the surge time T , due to the continuous clashing of coils. This must be considered a rather severe case of impact loading. Nevertheless, during the expansion phase of the spring, the originally very pronounced wave head can no longer be distinguished and the coils share rather equally in the total deflection.

4. Discussion of TD-record 70-728 (low without preload).

Operating conditions:

Here the impact velocity of the mass has been greatly reduced to

$$v_0 = 317 \text{ inch/sec.}$$

Response of spring:

Here the spring is more efficient in decelerating the slower mass and does not come quite so close to solid height:

$$H_{\min} = 5.95 \text{ inches.}$$

The surge times are found to be about the same as with the higher velocity level:

$$T = 12.05 \text{ ms}$$

$$\text{and } T_1 = .160 \text{ ms/coil.}$$

When the surge wave reaches the front end of spring again $t = 2T$, the superposition of 3 waves is fully absorbed by the spring, without clashing of coils. After $t = 3T$, the rear coils appear to become solid, with very minor clashing. The wavehead can be distinguished up to $t = 4T$, though quite faintly only.

5. Discussion of TD-record 70-730 (low with preload).

Here the spring is again preloaded to a height of about 13" and is struck by the impact mass at a velocity

$$v_0 = 322 \text{ inch/sec.}$$

Response of spring:

The spring reaches a minimum height $H_{\min} = 6.15"$, above solid height. Up to $t = 2T$, i.e. 2 wave superpositions, the coils do not clash and the wavehead evidently moves at constant velocity along the spring wire. After $t = 2T$, when the wavehead reaches again the front end of spring, 3 waves superpose to the initial deflection f_1 due to preload and this deflection cannot be absorbed anymore within the available clearance between coils. Rather, the front coils clash violently and this clashing is propagated toward the rear end of spring at much greater speed than within the first $2T$ sec. Notice the almost vertical front of the wavehead after $t = 2T$. However, after $t = 3T$, the wavehead cannot be discerned anymore.

G. 3-strand SPIW drive spring:Discussion of TD-records on sudden release of spring.1. Review of TD-records delivered.

A total of 6 TD-records was taken and delivered as follows:

Operating condition		No. of record in report	No. of record in second set
Preload	Support of rear end		
high	rearward only	70-734*	733*
medium	rearward only	735*	736*
medium	both ways (fastened)	738*	737*

The asterisk * denotes that these records were recorded on film rather than on paper. Therefore, no explanatory lines are shown in the prints, since they are not feasible on the film.

2. Discussion of TD-record 70-734*.(high)Operating conditions:

Here the spring was precompressed to a height $H_2 = 7$ inches, i.e., not far from the solid height H_s of about 5.5". The pitch of the 60 middle coils was measured on the spring to be $L = 15.64$ ", or a pitch

$$h_2 = .094 \text{ inch.}$$

This establishes the displacement magnification $q = .395x$.

Response of spring:

Upon the sudden release of the front end of spring, the latter expands with a practically constant velocity, which is measured to be:

$$v_0 = 832 \text{ inch/sec.}$$

Coil after coil follows in the release and the rear end of spring lifts off its rear one-way support after a time

$$T_s = 12.2 \text{ ms.}$$

By theory, this separation time T_s is equal to the surge time T . The rear end also reaches a maximum expansion velocity of about 850 inch/sec, but its velocity oscillates slightly about a mean value of roughly 830 inch/sec. Again, the originally step wave head broadens to a more and more wider and lesser slope, as seen in the record. This smoothening of the wavehead also accounts for the disturbance superposed to the expansion pattern of the spring coils at constant velocity.

3. Discussion of TD-record 70-735* (medium).

Here the spring is precompressed to a height of about .2 inches. More accurately, the 60 middle coils have an average pitch at pre-compression

$$h_2 = .161 \text{ inch/coil} .$$

Response of spring:

The front expands suddenly with a rather constant velocity

$$v_0 = 416 \text{ inch/sec} .$$

The release is propagated through the spring evidently at constant velocity and the rear end of spring lifts off its support after

$$T_s = 12.4 \text{ ms} .$$

4. Discussion of TD-record 70-738* (medium, rear end fixed).

Here the rear end of spring is fastened to the inner guide rod, rendering about one coil inactive. The spring then was precompressed to about the same height of 12 inches.

Response of spring:

The front end of spring performs a continuous vibration, extending at constant velocity over 2 surge times and then compressing over 2 surge times, etc.. The entire spring performs a vibration with the period

$$4T = 48.8 \text{ ms} ,$$

indicating a surge time $T = 12.2 \text{ ms} .$

At multiples of the surge time after spring release, the spring coils have the same potential and also the same velocity, in the following pattern:

Instant of time	Velocity	Potential
$t = 0$	0	in compression
$t = T$	in extension	0
$t = 2T$	0	in extension
$t = 3T$	in compression	0
$t = 4T$	0	in compression

and this cycle repeats itself, with somewhat decreasing amplitude.

H. 7-wire SPIW drive spring:Discussion of TD-records on impact loading.1. Review of TD-records taken.

Time-displacement records on the 7-wire SPIW spring are numbered from 70-741 to 70-760. The records on impact loading number from 741 to 750 as follows:

Operating conditions		No. of record	no. of record
Impact velocity	Precompression	in report	in second set
high	none	744	743
high	yes	746	745
low	none	748	747
low	yes	749	750

All these records on photorecording paper.

Note : It was found rather difficult to obtain strong enough traces over the entire length of spring, especially for the end coils. This is chiefly due to the configuration of the strand: The individual wires are rather thin and also give a favorable light reflection over a small longitudinal section only. Therefore, a stranded wire spring must be expected to give poorer TD-traces than a single wire spring of the same size. This deficiency has led to have some of the records recorded on very fast film rather than paper, e.g., all the records on sudden release of spring and also some on impact loading.

2. Discussion of TD-record 70-744 (high, no preload).Operating conditions:

Here the spring had already set to a free height $H_f = 13.80"$ (from 14.26"), due to extensive preliminary testing. The free pitch h_f of the 50 middle coils had been reduced to .231", from .238". As with the 3-strand SPIW drive spring, an impact mass of 1.15 lbs was used and the additional rubber stop was contacted by the mass at the same height $H = 7"$. The impact velocity was measured to be :

$$v_0 = 480 \text{ inch/sec.}$$

Response of spring:

The spring is being compressed to $H_{\min} = 5.6"$, i.e., close to the solid height of about 5.2". The main deceleration is provided by the additional rubber buffer, between the heights $H = 7"$ and 5.6".

The acceleration of the spring coils, which is instant-like for the front coil, spreads over a longer time interval, as the transition of wave progresses. The time T_h is defined as the transition time of the head of the wave, as far as it can be discerned in the record:

$$T_h = 9.0 \text{ ms}.$$

Likewise, the surge time per coil, T_1 , is determined using the head of the wave:

$$T_1 = .158 \text{ ms/coil}.$$

On the other hand, the surge time T is measured to the point where the wave has reached a spring end with its full amplitude and therefore is found to be somewhat larger than T_h :

$$T = 9.60 \text{ ms}.$$

Therefore, due to the smoothening wavehead in transition, there are different interpretations of the surge time of the wave.

The front coils clash after $t = 2T$, when 3 waves superpose.

3. Discussion of TD-record 70-746 (high, with preload).

Here the spring is precompressed at a height $H_1 = 11"$ (from 13.8") and is struck by the impact mass at a velocity $v_0 = 485"/s$.

Response of spring:

Again, the spring is being compressed to $H_{\min} = 5.6"$, close to its solid height, with the rubber buffer doing the essential stopping of the impact mass.

During the first transition of wave, from the front to the rear, a surge time

$$T = 9.0 \text{ ms} \quad \text{is measured.}$$

The head of the wave (only faintly discernible), however, travels somewhat faster, requiring a transition time

$$T_h = 7.5 \text{ ms only.}$$

Likewise, the surge time per coil T_1 (based on the progression of the wavehead) is found to be quite less than with the free spring:

$$T_1 = .139 \text{ ms/coil}.$$

Therefore, it appears that the preload promotes the smoothening out of the wavehead and raises its surge velocity.

After $t = T$, the rear coils start to clash. Evidently, the superposition of 2 waves to the preload exceeds the clearance between coils.

4. Discussion of TD-record 70-748 (low, no preload).

Here the spring is struck by the impact mass at its free height $H_f = 13.8"$ with a velocity $v_o = 315$ inch/sec.

Response of spring:

The spring reaches a minimum height $H_{min} = 6"$, in the range of the additional rubber buffer, which still performs the major work in stopping the impact mass.

From the first transition of wave, the following surge times were read:

$$T = 9.85 \text{ ms} ;$$

$$T_1 = .163 \text{ ms/coil} .$$

Up to the time $t = 3T$, the wave progresses through the spring without the clashing of coils and the surge times for the first, second and third passage through the spring are practically the same. After $t = 3T$, the rear coils show minor clashing; therefore, the superposition of 4 waves cannot be absorbed anymore by the original clearance between coils. However, all coils share rather equally in the total deflection of spring and no definite wavehead can be found after $t = 3T$.

5. Discussion of TD-record 70-749 (low, with preload).

Here the spring is again preloaded to a height $H_1 = 11"$ (from 13.8") and is struck by the impact mass of 1.15 lbs at a velocity

$$v_o = 335 \text{ "/s} .$$

Response of spring:

Here the end coils are partly hidden, due to some change in the position of the illuminating lamp, for stronger traces of the more important front coils.

The following surge times are read from the first transition:

$$T = 9.4 \text{ ms} ;$$

$$T_1 = .143 \text{ ms/coil} .$$

These times appear to be slightly shorter than without preload of Spring (TDR 70-743 above).

The second transition of wavehead takes about the same time T , without clashing of coils. However, after $t = 2T$, the front coils come close to clashing. But no wavehead can be discerned after $t = 2T$.

I. 7-wire SPIW drive spring:

Discussion of TD-records on sudden release of spring.

1. Review of TD-records taken.

A total of 6 TD-records was taken and delivered as follows:

Operating conditions		No. of record	No. of record
Preload	Support of rear end	in report	in second set
high	rearward only	70-751*	753*
medium	rearward only	755*	756*
medium	fixed both ways	759*	760*

All records taken on fast film. Therefore, no explanatory lines could be drawn on the original.

2. Discussion of TD-record 70-751* (high, one-way support).

Operating conditions:

These tests were taken before the impact tests, with the spring still at its original free height, $H_f = 14.26"$.

From the 50 middle coils, the average pitch at free height was measured to be:

$$h_f = .238"$$

The spring was then precompressed to a height $H_1 = 6.5"$, i.e., an average pitch of the 50 middle coils

$$h_2 = .107"$$

This makes for a deflection per coil

$$f_2 = .131"$$

Notice the higher film speed $p = 392 \text{ "/s}$.

Response of spring:

Upon the sudden release of the front end of spring, the latter expands at a constant velocity

$$v_o = 811 \text{ "/s}$$

Coil after coil follows in the release and the rear end of spring lifts off from its one-way support at a time

$$T_s = 9.9 \text{ ms}$$

after release, which theoretically equals the surge time T .

The head of the release wave (extension wave), however, traverses somewhat faster through the spring, requiring only a time

$$T_h = 7.8 \text{ ms}$$

(Notice third coil from rear).

3. Discussion of TD-record 70-755* (medium, one-way support).

Here the spring is precompressed to a height $H_2 = 10"$, or, more accurately, to an average pitch $h_2 = .167"$.

This means a deflection per coil $f_2 = .238 - .167 = .071"$.

Response of spring:

Here the camera shutter closed rather early, but still the most important time phase including the lift-off of the spring from the rear support was covered.

Extension velocity of front end of spring:

$$v_o = 432 \text{ inch/sec.}$$

Separation time of rear end from support:

$$T_s = T = 10.2 \text{ ms.}$$

Propagation time of wavehead through spring (see third coil from rear):

$$T_h = 9.0 \text{ ms.}$$

Here the two times T_s and T_h are relatively closer together than with the previous case of spring release from high preload.

4. Discussion of TD-record 70-759* (medium, rear end fixed).

Here the rear end of spring was fastened to the inner guide rod, rendering about one coil inactive. The spring was precompressed to an initial height H_2 of about $10"$.

Response of spring:

The entire spring performs a periodic vibration with the period

$$4T = 40.8 \text{ ms,}$$

or a surge time $T = 10.2 \text{ ms.}$

The spring coils undergo alternating phases of

no stress	,	velocity in extension
extension stress	,	velocity 0
no stress	,	velocity in compression
compression stress	,	velocity 0, etc..

The front end of spring extends and compresses at constant velocity, with its TD-trace being of saw-tooth shape. This is definitely not a harmonic or sinusoidal vibration of spring, but a special mode including many higher harmonics (see Fourier series).

J. 3-strand M85 drive spring :

Discussion of TD-records on impact loading.

1. Review of TD-records taken.

Time-displacement records on the 3-wire M85 drive spring are numbered from 70-761 to 70-780, with the following records being delivered:

Operating conditions		No. of record	No. of record
Impact velocity	Precompression	in report	in second set
high	none	764*	763
high	yes	767*	766
low	none	769	768
low	yes	772*	770

Note : A record number with * denotes a record on film rather than on photo-recording paper.

2. Discussion of TD-record 70-764* (high, no preload).

With an impact velocity

$$v_0 = 473 \text{ inch/sec} ,$$

the spring reaches a minimum height $H_{\min} = 7.45"$, which is only 0.30" less than the height at buffer contact. Considering the highly non-linear force-deflection characteristics of the additional rubber buffer, it can be said that the impact is being stopped practically by the spring alone.

The surge time is measured to be $T = 7.95 \text{ ms.}$

The coils of the spring come very close to clashing after $t = 3T$ and $t = 4T$ (4 single surge waves superposed).

It should be stated here that this spring is coiled very steeply, having an uncorrected solid stress of more than 185,000 psi, or a velocity potential in excess of

$$u_s = 120 \text{ ft/sec or } 1440 \text{ inch/sec} .$$

3. Discussion of TD-record 70-767* (high, with preload).

In spite of the preload, the spring is found to stop the impact of the mass practically on its own, without the help of the additional buffer ($H_{\min} = 7.7''$ near buffer contact).

The spring coils come close to solid after $t = 2T$ and contact each other after $t = 3T$ (see rear coils). The wavehead is visible over a time phase up to $t = 3.5T$. From there on, all the coils share about equally in the total deflection of the spring.

The surge time is read to be $T = 7.9$ ms.

4. Discussion of TD-record 70-762 (low, no preload).

With the lower impact velocity

$$v_0 = 382 \text{ inch/sec}$$

the spring is effective to stop the impact of the mass at a minimum height $H_{\min} = 9.35''$,
i.e., about 2" above solid height.

The coils do not clash and the wavehead is visible up to $t = 4T$. The surge time is measured to be

$$T = 7.80 \text{ ms.}$$

5. Discussion of TD-record 70-772* (low, with preload).

With an impact velocity

$$v_0 = 420 \text{ inch/sec, plus preload,}$$

the impact of the mass is stopped at a height

$$H_{\min} = 9.2'' ,$$

about 2" above solid height.

With preload, the wavehead disappears somewhat sooner, after $t = 3T$. Also, the surge time

$$T = 7.7 \text{ ms}$$

is measured to be somewhat shorter than without preload.

Note : It should be understood, that the impact velocity v_0 of the mass could not be evaluated to highest accuracy standards, due to the shortness of the available trace. Also, the recording edge of the mass is in a different optical plane than the coils. Therefore, the actual impact velocity may be 1 or 2 % less than given.

K. 7-strand M85 drive spring:

Discussion of TD-records on impact loading.

1. Review of TD-records taken.

The TD-records on impact loading are numbered from 70-781 to 70-792, with the following records being delivered:

Operating conditions:			No. of record	No. of record
Impact velocity	Precompression	Buffer	in report	in second set
high	none	none	782	781
high	none	yes	783*	---
high	yes	yes	786*	785
low	none	yes	787	788
low	yes	yes	790	791

2. Discussion of TD-record 70-782 (no preload, no buffer).

In this recording as well as in a few previous loadings, the impact mass failed to engage the buffer, so that the spring had to absorb the energy of impact alone. As a result, the spring took a substantial set from its original free height $H_f = 18.25"$. Prior to this recording, the spring had a free height $H_f = 17.25"$.

With an impact velocity of $v_0 = 490$ inch/sec, the spring is being crushed solid to

$$H_{min} = 5.4" ,$$

which is somewhat less than the static solid height of about 6".

Notice the minor reduction of compression velocity over the spring stroke. Only in the last $1\frac{1}{2}$ inch of spring stroke near solid height, the essential deceleration of the impact mass takes place. In other recordings with buffer engaged, the deceleration phase extends over more than 1 inch.

After $t = 2T$, when 3 amplitudes of surge wave superpose, the front coils start to clash. Notice the much greater velocity of propagation of the wave head from then on, which is about 3 times the normal propagation velocity, with clearance between coils.

The surge time through the spring was measured to be

$$T = 10.8 \text{ ms.}$$

3. Discussion of TD-record 70-783* (high, no preload, but with buffer).

Here the free height had been further reduced by setting, to 16.9".
 With an impact velocity $v_0 = 481$ inch/sec ,
 the spring comes down to $H_{\min} = 6.1"$,
 which is about the solid height of the spring in static testing.
 The main deceleration of the impact mass is performed by the additional buffer, which is compressed over 1.65" stroke, from $H = 7.75"$ to $H_{\min} = 6.1"$. Notice the more gradual deceleration of the spring end in this record as compared to TD-record 70-782 without buffer.

Again, the coils start to clash after $t = 2T$, when the amplitudes of 3 waves superpose.

The surge time per coil of the wave head (first transition) was measured to be

$$T_1 = .235 \text{ ms/coil} ,$$

i.e., about the same as in TD-record 70-782.

The entire surge time through the spring is found to be:

$$T = 11.7 \text{ ms.}$$

4. Discussion of TD-record 70-786* (high, with preload).

Here the spring was precompressed from a free height $H_f = 16.90"$ to an initial height $H_i = 14.5"$.

With an impact velocity $v_0 = 474$ inch/sec ,
 the spring coils at the rear end go practically into contact after $t = T$, but without clashing. The time for the second transition of the wave head is slightly less than the time for the first transition, indicating coil contact here and there.

The wavehead returns to the front end of spring near the end of the compression stroke and then the front coils clash, as indicated by the much steeper wave front, which is visible up to about $t = 2.5.T$. Again, the spring reaches practically solid height, with $H_{\min} = 6.05"$.

During the first transition, the surge time per coil of the wave head is measured to be $T_1 = .209$ ms/coil .

This value is distinctly less than for the records without preload .

The surge time for the entire spring is measured to be:

$$T = 10.8 \text{ ms.}$$

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5. Discussion of TD-record 70-787 (low, no preload).

Here the impact velocity was reduced to

$$v_0 = 396 \text{ inch/sec.}$$

With $H_{\min} = 6.40''$, the spring does not quite reach solid height. Here the wavehead returns to the front end of spring prior to buffer contact, when the compression velocity is still substantial. After $t = 2T$, some of the coils clash slightly during the third transition of surge wave, as indicated by the shorter transit time. The wave head is visible up to $t = 4T$.

During the first transition, the surge time per coil is found to be:

$$T_1 = .248 \text{ ms/coil.}$$

The entire surge time through the spring is measured to be:

$$T = 11.1 \text{ ms.}$$

6. Discussion of TD-record 70-790 (low, with preload).

Here the spring was precompressed to $H_1 = 14.5''$,
from a free height $H_f = 16.9''$.

With an impact velocity of 415 inch/sec, the spring reaches a height $H_{\min} = 6.40''$, slightly above solid.

After $t = 2T$, the front coils clash, as indicated by the much steeper wave front. The wavehead is visible up to $t = 3T$.

During the first transition, the surge time per coil of the wave head is $T_1 = .223 \text{ ms/coil}$,

which is again less than the corresponding value under no preload.

The entire surge time was measured to be :

$$T = 10.5 \text{ ms.}$$

L. 7-strand M85 drive spring:

Discussion of TD-records on sudden release of spring.

1. Review of TD-records taken.

A total of 6 TD-records was taken and delivered as follows:

Operating conditions:		No. of record	No. of record
Preload	Support of rear end	in report	in second set
high	rearward only	793*	794*
medium	rearward only	795*	796*
medium	fixed both ways	797*	798*

All TD-records on fast film.

2. Discussion of TD-record 70-793*.

Here the spring was precompressed to an initial height

$$H_2 = 9 \text{ inches ,}$$

as compared to a solid height of 6.2". More accurately, the level of precompression is measured by the pitch of the coils, which was found to be

$$h_2 = .202" \text{ over 40 coils.}$$

Here the spring had still the original free length $H_f = 18.26"$, or

$$h_f = .398" \text{ per coil ,}$$

since these sudden release tests were conducted prior to the impact loading tests. This means an initial deflection per coil

$$f_2 = .398 - .202 = .196" .$$

The front end of spring (released end) is found to extend at a constant velocity

$$v_0 = 796 \text{ inch/sec.}$$

The rear end lifts from its one-way support at a time

$$t = T_s = 11.55 \text{ ms}$$

after the sudden release of the front end and then also assumes a constant extension velocity of same amount v_0 .

The spring then bounces out between two stops. The superposed disturbance originating from the rear end at time $t = T_s$ is due to the wave head flattening out from its original step-form.

3. Discussion of TD-record 70-795*.

Here the spring was released from an approximate height

$$H_2 = 12 \text{ inches} .$$

More accurately, the level of preload was measured by the pitch of the precompressed coils: $h_2 = .266''$ over 40 coils.

With a pitch at free height $h_f = .398''$, the deflection per coil in the precompressed state then follows to be:

$$f_2 = .398 - .266 = .132 \text{ "/coil} .$$

The extension behavior of the spring is the same as when released from a higher preload. Here a constant extension velocity

$$v_0 = 543 \text{ inch/sec}$$

was measured.

The separation time T_s of the rear end was found to be:

$$T_s = 11.2 \text{ ms} .$$

4. Discussion of TD-record 70-797*.

Here the first and second end coil (this spring has open ends) were clamped to the guide rod, so the number of active coils was reduced by at least one coil to about 44 coils.

Here the pitch in the precompressed state was

$$h_2 = .263 \text{ "/coil over 40 coils} ,$$

i.e., an initial deflection $f_2 = .135''$.

The free end of spring is found to extend with a constant velocity

$$v_0 = 562 \text{ inch/sec} .$$

The spring performs a periodic vibration with the period

$$4T = 49 \text{ ms} ,$$

$$\text{or } T = 11.25 \text{ ms} .$$

M. 7-strand 20 mm gun drive spring:Discussion of TD-records on impact loading.1. Review of TD-records taken.

These TD-records on impact loading are numbered from 70-701 to 70-720, with the following records being delivered:

Operating conditions:		No. of record	No. of record
Impact velocity	precompression	in report	in second set
high	none	709*	710*
high	yes	712*	711*
low	none	714*	713*
low	yes	717*	716*
low	yes		715*

TDR 70-715* is an odd record, where the precompressing means of spring was released slightly prior to the actual impact of the mass.

Note : All the records delivered were taken at rather low magnification

$$q = .229x, \text{ or } 1:4.37 \text{ reduction,}$$

in order to accomodate the entire length of spring ($H_p = 29.3"$) within the film width of 8". Therefore, fast film had to be used instead of conventional photo-recording paper.

2. Discussion of TD-record 70-709*(high, no preload).

With a heavier impact mass of 3.25 lbs and an impact velocity

$$v_0 = 535 \text{ inch/sec,}$$

the spring reaches a minimum height

$$H_{\min} = 11.6"$$

as compared to a solid height of about 8.5". Therefore, this strong spring is capable of stopping the impact mass of its own and no additional buffer was installed. ✓

When the wave head reaches again the front end of spring at time $t = 2T$, the compression velocity is already much reduced and the third wave amplitude superposed is much less than that of the wave head. The coils come close to solid, but do not clash, as indicated by the third transit time being of normal length. After $t = 3T$, however, the rear coils clash, as shown by the much steeper wave front. The wave head is visible up to $t = 4T$.

During the first transition, the surge time per coil of the wavehead was found to be $T_1 = .455 \text{ ms/coil}$, with the entire surge time $T = 17.6 \text{ ms}$.

3. Discussion of TD-record 70-712* (high, with preload).

Here the spring is preloaded to a height $H_1 = 24.1"$, as compared to its free height $H_f = 29.1"$ (5" deflection).

With an impact velocity $v_o = 554$ inch/sec ,
the spring reaches a height $H_{min} = 11.3$ inches .

A few rear coils clash already after $t = T$ and the front coils clash after $t = 2T$, when 3 wave amplitudes superpose to the preload. The wave head is visible up to $t = 3T$.

The surge time is measured to be: $T = 17.5$ ms .

4. Discussion of TD-record 70-714* (low, no preload).

Here the impact velocity has been reduced to

$$v_o = 402 \text{ inch/sec.}$$

Here the spring is found to stop the impact mass at a minimum height

$$H_{min} = 16.3 \text{ inches ,}$$

which is way above its solid height of about 8.5 inches.

The coils never clash during their response to impact and the wavehead is visible up to $t = 3T$.

The surge time is measured to be $T = 18.1$ ms.

5. Discussion of TD-record 70-717* (low, with preload).

Here the spring has been precompressed to a height

$$H_1 = 20.4 \text{ inches ,}$$

i.e., a considerable initial deflection $F_1 = 8.7"$ from its free height of 29.1".

With an impact velocity $v_o = 408$ inch/sec,
the spring reaches a minimum height $H_{min} = 14.3$ inches.

The wavehead reaches the front end of spring at time $t = 2T$, when the compression velocity is about 0. No clashing of coils occurs and the wavehead is not visible after $t = 2T$.

The surge time is read to be $T = 16.9$ ms ,
i.e., distinctly less than in record 70-714* without preload.

N →

N. Attenuation of single surge wave in stranded wire spring.

The impact of a mass on a spring generates a single surge wave with an infinitely steep wavehead. Both the velocity v and the potential u of the end coils jump suddenly from 0 to a certain value v_0 .

Diagram 4 on page Y8 of Appendix E illustrates the velocity distribution $v(x,t)$ over the length L of the spring at various instants of time $t = 0, T/4, T/2, 3/4 T$, and $t = T$. The potential distribution $u(x,t)$ is similar. This diagram illustrates in a general manner the change of the wave shape during the first transition through the spring, as shown by all the impact records taken. This first transition of the impact wave through the spring is best suited to study the basic attenuation behavior, since the wave propagation is not disturbed or camouflaged by wave superpositions as yet.

At time $t = 0$, the wavehead is entering the spring with a vertical step, as indicated by the dotted line $v(x,0)$.

At time $t = T/4$, the peak of the wave has reached the quarter point $x = L/4$, while the wavehead is already at $x = .275 L$ and the step has changed into a steep slope. The spring coils within this slope are still being accelerated.

At time $t = T/2$, the wave peak has reached the half point $x = L/2$. The head of the wave is at $x = .55 L$, the slope of the wavehead has decreased and the wavehead comprises now more coils still being under acceleration.

As the wave progresses further, the wave head comprising all the coils still under acceleration gets longer and longer, as the distributions at $t = .75 T$ and T show. The beginning of the wavehead has a shorter surge time than the wave peak, e.g., in Diagram 4:

$$T_h = .91 \cdot T$$

Therefore, we must clearly distinguish between the surge time of the wave peak (T) and the surge time of the wavehead (T_h). The same applies to the surge times per coil, T_1 and T_{1h} . The surge time per coil T_1 listed on all the records and evaluated therefrom is actually the surge time of the wavehead, but the second subscript was omitted for simplicity reasons.

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The smoothening-out of the wave front during propagation is typical for all stranded wire springs. This behavior has already been observed previously on single wire springs, but to a much lesser degree. It is caused by the damping inherent in the spring and is a welcome feature, since it aids in distributing uneven impact loads more and more evenly over the length of spring, so that all the coils share about equally well in the total deflection of the spring.

Behavior of surge waves after the first transition:

After the first transition, several single surge waves superpose in the spring. However, the wavehead is visible in the TD-records over a length of time from $3T$ to $4T$. Under preload, the wavehead disappears faster, after from $2.5T$ to $3T$.

For single wire springs it has been found previously, that the wavehead is discernible over 6 to 8 surge times T , due to the lesser damping inherent in these springs.

On the other hand, it might be mentioned here, that in highly damped ring springs the wavehead can only be distinguished up to about $1.5T$. Therefore, with respect to wave attenuation, stranded wire springs lie somewhere in the middle between single wire springs and ring springs.

Quantitative measurement of damping:

The above statements about the attenuation of surge waves in stranded wire springs are largely of a qualitative nature. In order to arrive at a quantitative criterion or indicator, standard tests should be defined for determining the degree of damping: E.g., for a suddenly released spring with rear end fixed, the decay of the amplitudes in compression and extension could be measured and a damping constant could be derived therefrom. However, such standard tests and methods are not established as yet, not even for highly damping spring types as the ring spring.

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0. Comparison of measured and calculated surge times T.

One of the major objectives of this study is to obtain realistic formulas for stranded wire springs and to establish the difference in dynamic response as compared to single wire springs. For the latter, a complete set of formulas is available since 20 years and these formulas were well proven by many TD-recordings under a variety of operating conditions.

Calculation of surge times T :

Table II on page Y2 of Appendix Y lists the surge times T of the 5 test springs as calculated under two different assumptions:

Assumption A : The spring rate R is the algebraic sum of the rates of the component single wire springs.

Assumption B : The "initial spring rate" of the measured force-deflection diagram P(F) [Linear portion] has been used.

According to formula (26), the surge time T increases in inverse proportion to the square root of the spring rate R :

$$T \approx 1/\sqrt{R}$$

Therefore, it must be expected, that the surge time T will decrease with increasing levels of spring compression.

Measured values of surge time T :

Table III on page Y3 of Appendix Y lists the operating conditions for 40 TD-recordings of impact loading of the 5 test springs. These data show the intensity of impact in terms of the initial deflection per coil, f_1 , and impact velocity v_0 .

The measured surge times from these 40 impact loadings as well as from 18 sudden release tests are compiled in Table IV on page Y4 of Appendix Y.

These measured data have also been plotted in Chart I on page Y5 of Appendix Y, for best visual illustration of the results. This Chart is well suited to determine the influence of various parameters, e.g.,
level of impact velocity, v_0 ,
preload versus no preload,
sudden release tests versus impact loading.

Influence of the impact velocity level on surge time:

It appears, that in most cases the surge time T is slightly greater with the lower impact velocity than with the higher one. However, this is not absolutely conclusive, since the differences are quite small and also erratic. This uncertainty is quite plausible, since the 2 levels of impact velocity v_0 are not vastly different (see Table III for operating conditions). Recordings at rather different velocity levels, say 100 , 300 , and 500 inch/sec are believed to shed more light on the influence of v_0 on the surge time T .

Influence of the preload versus no preload:

Chart I shows clearly, that the surge times T decrease with increasing preload and compression level. These results confirm formula (26) : With increasing preload, the average spring rate also increases due to the progressive, non-linear force-deflection characteristics, and a shorter surge time T must be expected.

Sudden release tests versus impact loadings:

With the 3 springs tested, the surge time T in sudden release was always found to be a few percent higher than in impact loading. This result is also quite plausible: In extension, the direction of the frictional forces is reversed as compared to compression and a lower force-deflection characteristics $P(F)$, with a lower rate R' , results. According to formula (26), a longer surge time T is then to be expected.

Comparison of calculated and measured values of T :

In Chart 1, the T -value calculated under assumption A is indicated by a dotted line and the T -value resulting from assumption B by a solid line.

The T -values (A) agree well with the measured values for the two 3-wire strand springs, but are substantially greater than the measured values for the 3 7-wire strand springs.

The T -values (B) agree fairly well with the measured values for all 5 springs. Only with the SPIW-7 and M85-7 springs, the measured values with preload lie still below the calculated values and suggest, that a still higher spring rate than the initial rate should be used for closer agreement.

P. Comparison of measured and calculated extension velocities v_o in sudden release tests.

Again, the objective is to establish a formula to predict the extension velocity v_o in sudden release from the given precompression, for stranded wire springs in particular.

Measured values of extension velocity:

For 3 test springs, 6 TD-records each were taken in sudden release of spring, with 2 records for each operating condition. It was found, that the extension velocities of these 2 records varied very little. In Table V on page Y6 of Appendix Y, the average extension velocity v_o evaluated from these 2 records is listed. The range is

from 416 to 831 inch/sec .

Furthermore, the initial deflection per coil, f_2 , of the precompressed springs is also given in Table V:

Calculated values of extension velocities:

Here the formula
$$v_o = f_2/T_1 \quad (28a)$$

was used. T_1 depends on the spring rate R . Again, assumptions A and B are made and the T_1 -values obtained under these assumptions are also listed in Table V (top). Finally the v_o - values obtained from above formula (28a) are given in Table V.

Comparison of measured and calculated values.

In Table V, the ratios of measured to calculated velocity are also listed. Furthermore, for best visual illustration, the v_o -values have been plotted in Chart II on page Y7 of the Appendix Y.

With the 3-wire strand SPIW spring, the measured velocity v_o is about 5% less than the value calculated by either method, A or B.

With the two 7-wire strand springs, only the method B gives good agreement: Here the measured value v_o is 4.3% and 1.3% higher than the value calculated under assumption B. This latter result is quite plausible: Due to the progressive characteristics $P(F)$, the precompressed spring contains actually a higher potential energy than indicated by the initial spring rate R .



Q. Summary of major conclusions.

1. Organisation of Final Report.

The text of this report has been supplemented by data and illustrations in 3 Appendices:

Appendix X gives basic spring data in the form of data sheets and measured Force-deflection diagrams (given data).

Appendix Y gives the summary of both calculated and measured spring response data in both tabular and graphical form.

Appendix Z contains 30 samples of time-displacement records (altogether 60 records taken and evaluated) and because of its volume is submitted in a separate cover.

2. Comments on spring rates.

Stranded wire springs are found to have a non-linear, progressive force-deflection characteristics $P(F)$, as illustrated by the measured $P(F)$ -diagrams 1 to 3 in Appendix X.

It appears proper to distinguish between 3 force-deflection rates $R = P/F$:

- a) The measured initial rate, as defined by the linear initial portion of the diagram, over the first few inches of stroke.
- b) The measured average rate for a certain stroke extending well into the progressive portion of the diagram.
- c) The calculated rate, calculated under the simplifying assumption, that the total rate is the algebraic sum of the spring rates of the single wires in the strand.

From a comparison of these rates it was found, that even the measured initial rate is always higher than the calculated rate, showing an increase of a few percent for a 3-strand spring and increases up to 20% for a 7-strand spring. With large deflections close to solid height, the measured average rate is increased by still larger percentages.

3. Basic behavior of surge waves.

In general, stranded wire springs were found to respond to dynamic loading (impact loading or sudden release) in the same fashion as single wire springs. The surge waves generated travel through the spring at a constant surge velocity and also superpose in the same manner as known from single wire springs. However, there are also marked differences, as the next paragraphs will show.

4. Influence of the deflection or preload on the surge velocity.

In single wire springs with linear spring rate, the surge velocity c was found to be the same for all levels of spring compression. In the stranded wire springs tested, however, the surge velocity was found to rise slightly with higher loads and deflections. This result is quite plausible from the theoretical viewpoint: The theory predicts that the surge velocity increases in proportion to the square root of the spring rate R .

5. Smoothering of the wavehead.

In a stranded wire spring, the initially infinitely steep slope of a single surge wave generated by impact will decrease in transition and will spread over several coils by the time the wavehead reaches the rear end of spring. This response is due to the mechanical and friction coupling between the wires of the strand and is much more pronounced than in single wire springs.

Due to this behavior, the surge time of the wavehead (first response) is shorter than the surge time of the wave peak (where the wave has reached its full amplitude). Therefore, it becomes necessary to distinguish between these two surge times.

6. Disappearance of the wavehead.

In the TD-records on impact loading, the wavehead is visible over a time period of from $2T$ to $3.5.T$ after initiation. This time period is much shorter than with single wire springs of comparable length and surge time, where the wavehead is discernible up to $6T$ or $8T$ after initiation.

This behavior is caused by the pronounced smoothering of the wavehead during transition due to higher inherent damping. This response must be considered favorable and desirable, since it aids in making all the coils to share about equally in the total deflection of the spring. The peak stress is thus kept as low as possible, resulting in longer spring life.

7. Formula for surge time and surge velocity.

In the surge time formulas (11) and (12) for single wire springs, the wire diameter d and the mean coil diameter D enter as the major independent variables.

For stranded wire springs, it has been found more practical to use a less explicit form, because of the more complex configuration. In formula (26), the active weight W of the spring and its rate R enter as independent variables, rather than d and D . Here the question arises, which one of the 3 spring rates listed in Paragraph Q2 above should be used in order to arrive at realistic values for the surge time T .

This question was decided by the comparison of measured surge times (from the TD-records) with calculated surge times (using different values for R). It was found that the measured initial rate gives rather good agreement between measured and calculated surge times T . There is some indication, that sometimes a still higher value of R (toward the measured average rate) would improve the agreement. See Chart I in Appendix Y.

8. Formula for the extension velocity in sudden release.

According to formula (28) and (28a), the extension velocity v_0 is directly proportional to the precompression as expressed by the initial deflection per coil, f_2 , and inversely proportional to the surge time per coil, T_1 . Again the question arises, by which spring rate R this surge time per coil T_1 should be calculated. And again it is found, that fair agreement between measured and calculated values of v_0 is obtained by using the measured initial spring rate R . A higher spring rate toward the measured average rate would still improve this agreement.

9. Formula for induced load or force.

Here formula (29) applies. Judging from the above results, again the initial measured spring rate R should be used for close results. Of course, since no dynamic forces were measured, there was no direct comparison possible. However, when calculating the force from the measured deflection in the records, fair agreement was reached again.

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10. Comments on the time-displacement records.

It was found more difficult to obtain good recordings on these 5 stranded wire springs than on single wire springs recorded previously.

On the one hand, the thinner wires, which also lend a much shorter length of wire suitable for light reflection, must be expected to give a less intense and poorer image point than the coil of a single wire spring.

On the other hand, due to the great length of some of the springs, the recordings had to be made at reduced scale, from about .25x to .40x. In certain cases, especially with the 20 mm-7 spring and also in sudden release recording, fast film was used instead of the slower photo-recording paper.

In general, the records obtained show many fine details and the results evaluated from them confirmed the above drawn conclusions again and again. These records contain a wealth of information and may still be evaluated further with other objectives in mind.

Special attention was given to comprehensive and thorough reporting of the operating conditions for each record, to make the evaluated results most meaningful.

Furthermore, the evaluations of times, velocities, etc., were performed as accurately as possible, in order to bring out the sometimes small but still definite differences in dynamic response to that of single wire springs.

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R. Recommendations.

1. Influence of the compression level on surge velocity.

This influence could be checked out over the full compression range by specially devised test recordings:

Only 7-strand springs need be used, since they show the most progressive P(F)-diagram. Here the test spring would be precompressed to various ratios $f_1/f_s = 0, .25, .50, .75$ and the spring would be given a limited compression stroke by impact, of less than 2T duration (to avoid more than 2 wave superpositions). The impact velocity would be low, say about 120 inch/sec. These records would then be evaluated regarding surge time/coil of the wavehead and wave peak. The results would reveal the dependency of T and c over the full range of compression level.

2. Quantitative measurement of damping.

Special tests are to be devised and recordings made to establish a quantitative measurement of the damping inherent in a spring. E.g., a very light impact mass (as light as the spring or lighter) would be used and both the impact velocity and rebound velocity would be evaluated in order to determine the mechanical efficiency rate

$$\eta = v_0'^2 / v_0^2 \quad (32)$$

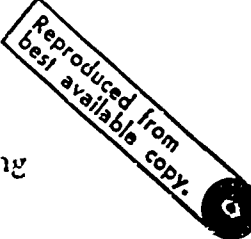
Or the percentage of actual damping to critical damping could be determined from a free vibration.

3. Attenuation of waves in other media.

In order to obtain a full qualitative understanding of the attenuation process in a linear medium and also to arrive at some simple formulas for quantitative damping, the propagation of impact waves should also be recorded for other damping springs and materials, such as a Poly-Urethane bar, a stack of rubber discs, a bar of hi-damp silicone, etc..

A P P E N D I X X

Basic spring data.



Page

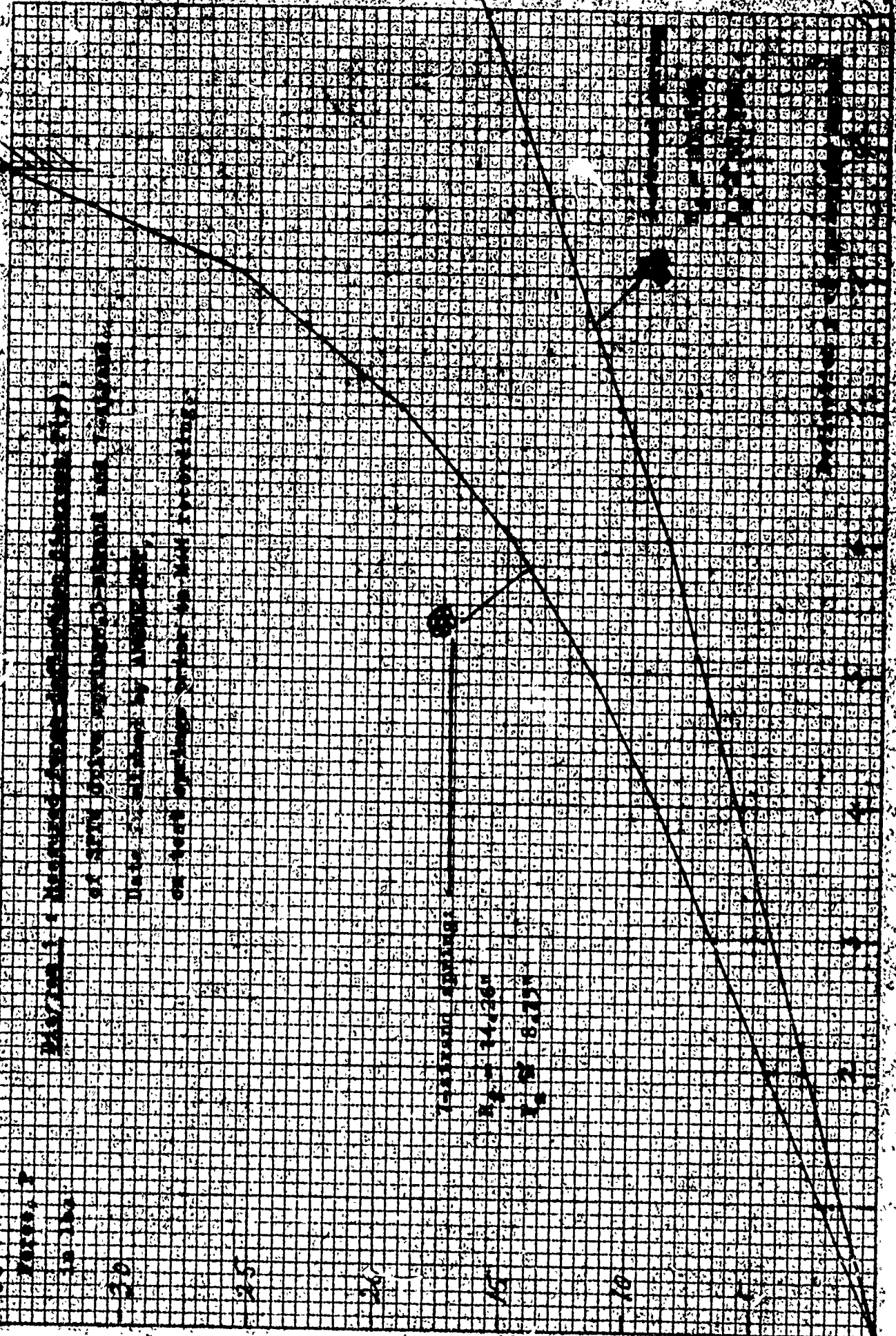
Data sheet on 3-strand SPIW drive spring	X1
Data sheet on 7-strand SPIW drive spring	X2
Diagram 1 : Measured force-deflection diagram P(F) on SPIW drive springs, 3- and 7-strand	X3
Data sheet on 3-strand M85 drive spring	X4
Data sheet on 7-strand M85 drive spring	X5
Diagram 2 : Measured force-deflection diagram P(F) on M85 drive springs, 3- and 7-strand	X6
Data sheet on 7-strand 20 mm gun drive spring	X7
Diagram 3 : Measured force-deflection diagram P(F) on 20 mm gun drive spring, 7-strand	X8

Basic data on SPIW drive spring, 3-strand wire.

Drawing No. :	AAI, No. 97384, B53099-10096.
Material	music wire
Wire size, d	.034
Strand diameter, d* (reference)	.073
Outer diameter of coils, D_o (measured)	.451
Inner diameter of coils, D_i (measured)	.317
Mean coil diameter, D	.384
Diameter of inner guide rod	.300
Index of coils, $C = D/d$	11.3
Type of spring ends	closed
Total number of coils, N_t	76.5
Number of active coils, N_a	74
Free height of spring, H_f	17.14
[H_f did not change during test program, no setting]	
Length of 60 middle coils ($60 \cdot h_f$) at free height H_f	13.86
Average pitch at free height, h_f	.231
Approximate solid height, H_s	5.7
Force deflection rate, P/F , in lbs/inch:	
Calculated from standard formula	1.38
Measured over the first 5" of stroke (also by drawing)	1.40

Basic data on SPIW drive spring, coiled from 7-wire strand.

Material	music wire, QQ470
Number of outer wires	6
Diameter of outer wires, d	.026
Diameter of core wire	.029
Diameter of wire strand (reference)	.0814
Outer diameter, D_o , measured	.442
Inner diameter, D_i , measured	.278
Mean coil diameter, D	.360
Diameter of inner guide rod in testing	.266
Spring index of outer wires, $C = D/d$	13.85
Type of spring ends	closed
Total number of coils, N_t (after testing)	60.5
Number of active coils, N_a	58
Free height H_f , before and after testing	14.26 / 13.80
Length of 50 middle coils, before and after testing	11.90 / 11.65
Average pitch at free height, h_f ,	.238 / .233
Approximate solid height, H_s	5.3
Force-deflection rate, P/F , in lbs/inch:	
Calculated from formula	1.85
Measured over the first 5" of stroke	2.10



10 X 10 PIN INCH

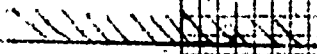
Basic data on M85 drive spring, coiled from 3-wire strand.

No drawing of spring, all dimensions measured.

Wire diameter, d		.065
Strand diameter, d^* , (reference)		.140 +
Outer diameter of coils, D_o (average)		.700
Inner diameter of coils, D_i		.420
Mean coil diameter, D		.560
Diameter of inner guide rod		.375
Index of coils, $C = D/d$ (husky spring!)		8.62
Type of spring ends		closed
Free height, H_f , before and after testing		18.30 / 18.20
Total number of coils (after testing), N_t		44.5
Number of active coils, N_a		42
Length of 40 middle coils	$40 \cdot h_f$	= 16.84 / 16.76
Average pitch at free height	h_f	= .421 / .419
Approximate solid height, H_s		6.8
Force-deflection rate, P/F , in lbs/inch:		
Calculated from formula		10.45
Measured over the first 5" of stroke		11.20

Basic data on M85 drive spring, 7-strand.

Spring drawing :	Springfield Armory B 7791254
Material	Spec MIL-W-13604 : FS 1085A
Number of outer wires	6
Diameter of outer wires, d	.038
Diameter of core wire, d	.041
Diameter of wire strand, d^* (reference)	.117 +
Outer diameter of coils, D_o (measured) after testing)	.650
Inner diameter of coils, D_i (measured)	.410
Mean coil diameter, D	.530
Diameter of inner guide rod	.375
Index of coils, $C \approx D/d$ (outer wires)	13.95
Type of spring ends	plain
Total number of coils, N_t (after testing)	45.4
Number of active coils, N_a	45
Free height, H_f , before and after recording tests	18.26 / 16.90 !
[Spring set considerably, partly due to impact compression to H_s]	
Length of 40 middle coils at H_f , before and after tests	15.92/15.04
Average pitch at free height, h_f	.398/.376
Approximate solid height, H_s	6.2
Force-deflection rate, P/F , in lbs/inch:	
Calculated from formula (sum of single wire springs)	3.29
Measured over the first 3" of stroke	4.0

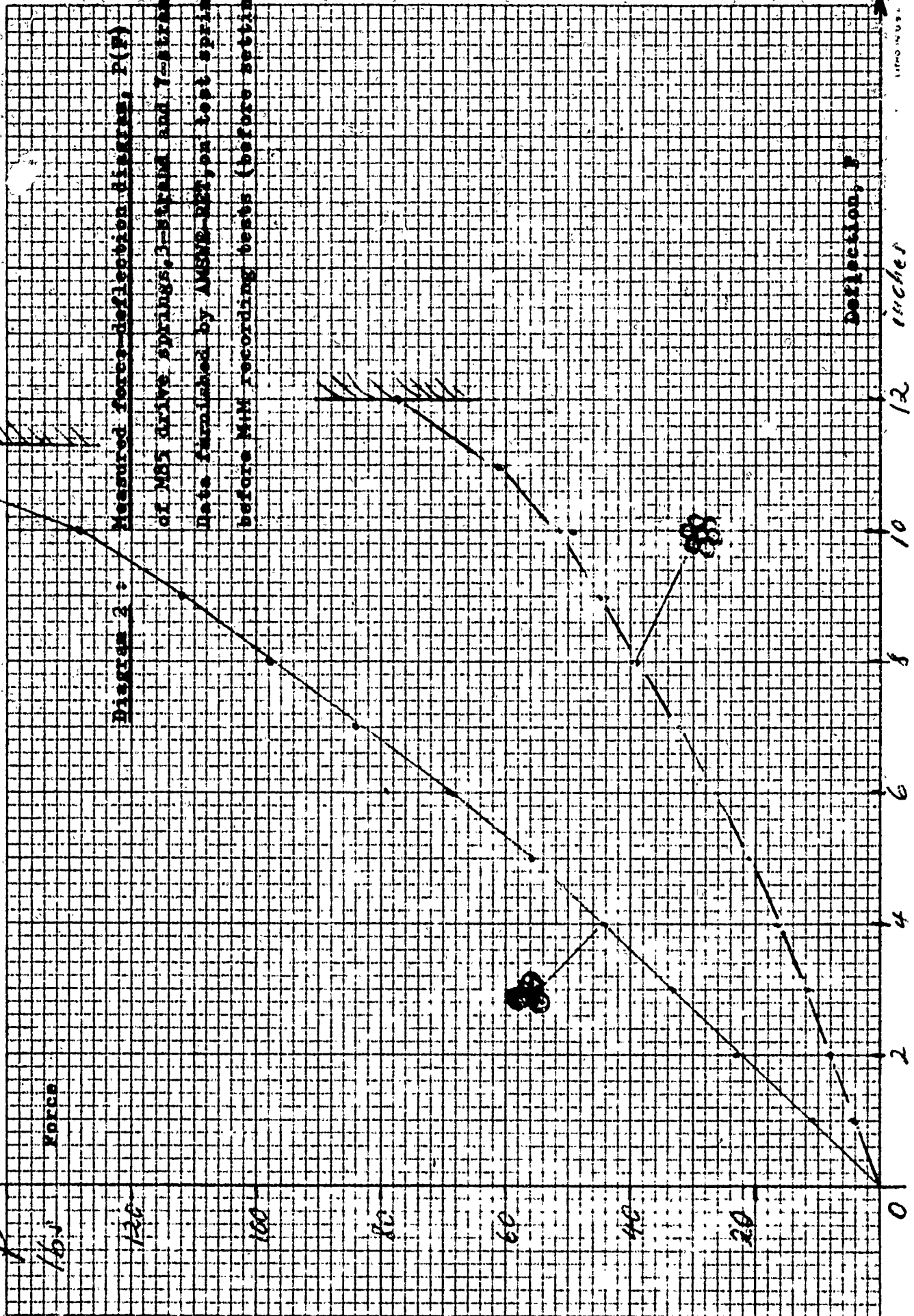


R-10
10 X 10 PER INCH

Force
160
120
100
80
60
40
20
0

Diagram 2: Measured force-deflection diagram, $P(\delta)$
of MBS drive springs, 3-standard and 7-altered.
Data furnished by AMSW-DET, on test springs
before M-M recording tests (before setting).

Deflection, δ
inches
12
10
8
6
4
2
0



Basic data on 20 drive spring, 7-strand:

No spring drawing furnished, all data measured.

Number of outer wires	6
Diameter of outer wires, d	.067
Diameter of core wire (estimated)	.072
Diameter of wire strand, d^* , reference	.206
Outer diameter of coils, D_o	1.205
Inner diameter of coils, D_i	.795
Mean coil diameter, D	1.000
Diameter of inner guide rod used in testing	.750
Index of coils, $C = D/d$ (outer springs, rather slender)	14.93
Type of spring ends	closed, plain
Total number of coils, N_t (after testing)	36.5
Number of active coils, N_a	34
Free height before and after testing, H_f	29.3
Free height durinh testing (slight temporary set)	29.1
Length of 30 middle coils, before and during testing	24.65/24.5
Average pitch at free height, h_f , before and during test	.822/.817
Approximate solid height, H_s	8.5
Force-deflection rate, P/F , in lbs/inch:	
Calculated from formula (sum of single wire springs)	6.25
Measured over the first 10" of stroke	7.50

LITHO IN 20 IN.

8-10
10 X 10 PER INCH

Diagram 1: Measured force-deflection diagram, P (P),

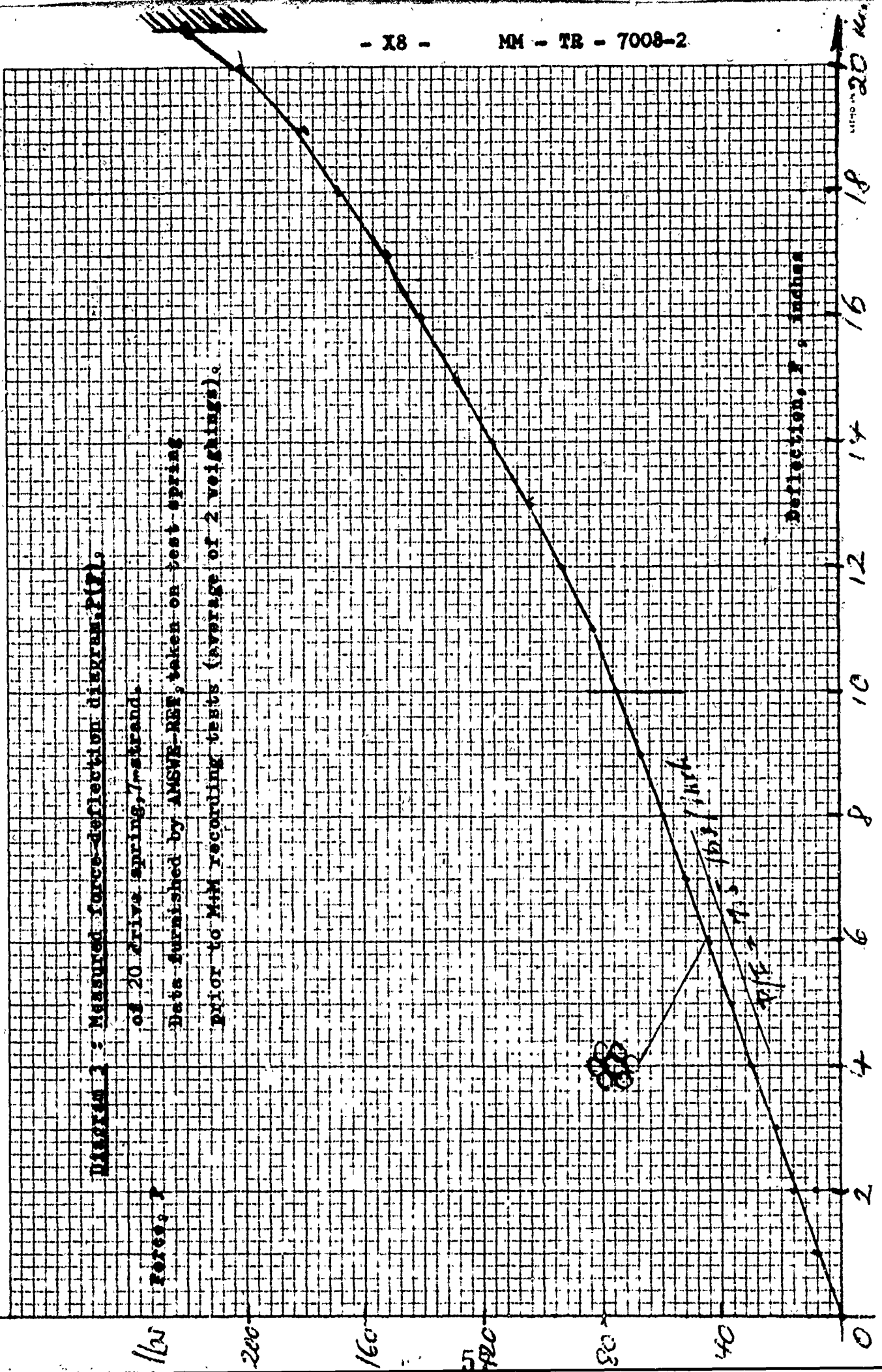
of 20 drive spring, 1/2 strand.

Data furnished by AMSVE-REF, taken on test spring

prior to M+M recording tests (average of 2 weighings).

Force, P

Deflection, δ , inches



A P P E N D I X Y

Additional Spring Data :

Data evaluated from the TD-records or calculated from Theory.

- Table I : Length of wire in coil, L_1 , for 5 test springs
- Table II : Numerical calculation of surge times and surge velocity based on two alternate assumptions
- Table III : Summary of operating conditions in impact loading tests
- Table IV : Comparison of surge times, measured and calculated
- Chart I : Surge time T , measured vs calculated values
- Table V : Sudden release tests:
Comparison of calculated and measured extension velocities
- Chart II : Extension velocity v_0 , measured vs calculated
- Diagram 4 : Attenuation of wavehead in first transition

Table I : Length of wire in coil, L_1 , for 5 test springs.

Spring	SPIW-3	SPIW-7	M85-3	M85-7	20mm-7
Wire diameter, d	.034	.026	.065	.038	.067
Mean coil dia, D	.384	.360	.560	.530	1.00
Index, D/d	11.3	13.85	8.62	13.95	14.93
Circumf., πD	1.204	1.130	1.757	1.662	3.142
Free pitch, h_f	.231	.238	.421	.398	.822
$1 + (h_f/\pi D)^2$	1.037	1.044	1.057	1.057	1.068
$\sqrt{1 + (\dots)^2} = \frac{L_1}{\pi D}$	1.018	1.022	1.028	1.028	1.034
This is relative length of coil helix centerline, compared to πD .					
Absolute length of coil helix CL	1.225	1.155	1.807	1.71	3.25
No. of turns/coil	10	16	7	15.5	17
length of lay, b	.367	.434	.773	.662	1.15
Radius of spiral, a	.0195	.0275	.0375	.0395	.0695
circumf., $2\pi a$.123	.170	.235	.248	.437
length ratio, b^*/b	1.054	1.076	1.045	1.068	1.07
Length L_1 of wire in one coil	1.290	1.243	1.89	1.823	3.48
length ratio $L_1/\pi D$	1.070	1.10	1.075	1.097	1.108

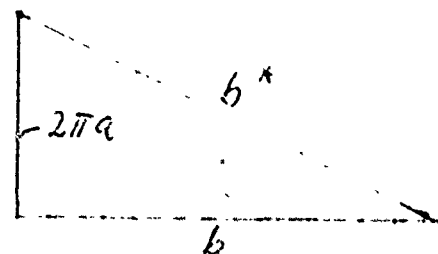
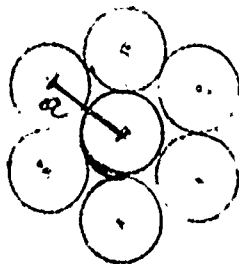
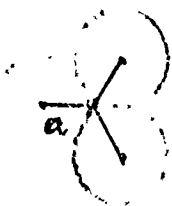


Table II : Numerical calculation of surge times and surge velocity, based on 2 alternate assumptions, R calculated or measured.

Spring	SPIW-3	SPIW-7	M85-3	M85-7	20mm-7
Wire diameter, d	.054	.026 .029	.065	.038 .041	.067 .071
Area A = .785.d ²	.00091	.00053 .00066	.00332	.00114 .00132	.00353 .00396
Area A of strand	.00273	.00384	.0100	.00816	.0251
Length of coil, L ₁	1.290	1.243	1.89	1.823	3.48
Weight of coil, W ₁	.00099	.00135	.00535	.00421	.0247
Number of active coils	74	58	42	.45	34
Active weight W = N.W ₁	.073	.078	.225	.1895	.84
First assumption: (A)					
Spring rate R <u>calcul.</u>	1.38	1.85	10.45	3.29	6.25
Rate per coil, R ₁	102	107.2	439	148	212.5
Surge time/coil, T ₁ ,ms	.1585	.181	.178	.272	.550
Surge time T = N.T ₁ ,ms	11.73	10.5	7.47	12.23	18.7
Surge veloc. c = L ₁ /T ₁	8,140	6,870	10,620	6,700	6,330
Force : $\Delta P/v = R_1 \cdot T_1$.01615	.0194	.078	.0402	.117
Second assumption: (B)					
Initial rate measured	1.40	2.10	11.20	4.00	7.50
Rate per coil, R ₁	103.6	121.7	470	180	255
Surge time/coil, T ₁	.157	.1695	.172	.246	.501
Surge time T = N.T ₁	11.6	9.83	7.20	11.06	17.05
Surge veloc. c = L ₁ /T ₁	8,210	7,340	11,000	7,400	6,940
Force: $\Delta P/v = R_1 \cdot T_1$.0163	.0206	.0805	.0442	.1275

Table III: Summary of operating conditions in impact loading tests.

(Number of TD-record, impact velocity v_0 , free pitch h_f , and f_1).

SPIW-3 W = 1.15 lbs		SPIW-7 W = 1.15 lbs		M85-3 W = 2.15 lbs		M85-7 W = 2.15 lbs		20mm-7 W = 3.25 lbs	
722	475	743	485	763	480	782	490	709*R	535
	.231		.232		.421		.384		.817
---	---	---	---	---	---	---	---	---	---
723R	478	744R	480	764*R	473	783*R	481	710*	553
	.231		.232		.419		.376		.817
---	---	---	---	---	---	---	---	---	---
725R	465	745	492	766	495	785	521	711*	564
	.231		.232		.419		.376		.817
	.055		.046		.061		.050		.191
726	465	746R	485	767*R	494	786*R	474	712*R	554
	.231		.232		.419		.376		.817
	.053		.047		.043		.056		.147
727	314	747	300	768	390	787R	396	713*	380
	.231		.232		.419		.376		.817
---	---	---	---	---	---	---	---	---	---
728R	317	748R	315	769R	382	788	403	714*R	402
	.231		.232		.419		.376		.817
---	---	---	---	---	---	---	---	---	---
730R	322	749R	335	770	359	790R	415	716*	400
	.231		.232		.419		.376		.817
	.056		.054		.074		.051		.253
731	308	750	325	772*R	420	791	409	717*R	408
	.231		.232		.419		.376		.817
	.055		.050		.045		.051		.256

781-no buffer }
782R-no buffer } 715*-odd record

TABLE IV : Comparison of surge times T_1 and T , measured and calculated.

SPIW-3		SPIW-7		M85-3		M85-7		20mm-7	
Rec.No.		Rec.No.		Rec.No.		Rec.No.		Rec.No.	
T	T_1	T	T_1	T	T_1	T	T_1	T	T_1
722		743		763		782R		709*R	
11.8	.159	9.4	.158	7.8	.170	10.8	.230	17.6	.455
723R		744R		764*R		783*R		710*	
12.1	.154	9.6	.158	7.95	.170	11.7	.235	17.7	.47
725R		745		766		785		711*	
12.0	.155	8.7	.141	7.47	.159	10.75	.219	17.1	---
726		746R		767*R		786*R		712*R	
11.6	.160	9.0	.139	7.9	.161	10.8	.209	17.5	---
727		747		768		787R		713*	
11.6	.157	9.9	.170	7.77	.163	11.1	.248	17.9	---
728R		748R		769R		788		714*R	
12.05	.160	9.85	.163	7.8	.160	11.1	.254	18.1	---
730R		749R		770		790R		716*	
11.8	.160	9.4	.143	7.8	.161	10.5	.223	17.2	---
731		750		772*R		791		717*R	
11.6	.159	8.65	.134	7.7	.152	10.6	.214	16.9	---
Avg. 11.8	.159	9.3	.151	7.77	.162	10.9	.229	17.5	.462
Cal. 12.6	.157	9.83	.1695	7.20	.172	11.06	.248	17.05	.501
Sudden release tests (some springs up to one coil shorter):									
733	12.03	751R	9.9	***-----		793*R	11.55	-----	
734R	12.2	753	10.2			794*	11.6		
735R	12.4	755R	10.2			795*R	11.2		
736	12.1	756	10.5			796*	11.3		
737	12.2	759R	10.2			797*R	11.25		
738R	12.2	760	10.2			798*	11.2		
Averages: 12.2		10.2				11.35			

Surge Time

Chart I

Surge Time

measured

20
18
16
14
12
10
8
6
4
2
0

high vol, with P
high vol, with P
low vol, with P
low vol, with P
sudden release

impact loading

SPIW-3

SPIW-7

M85-

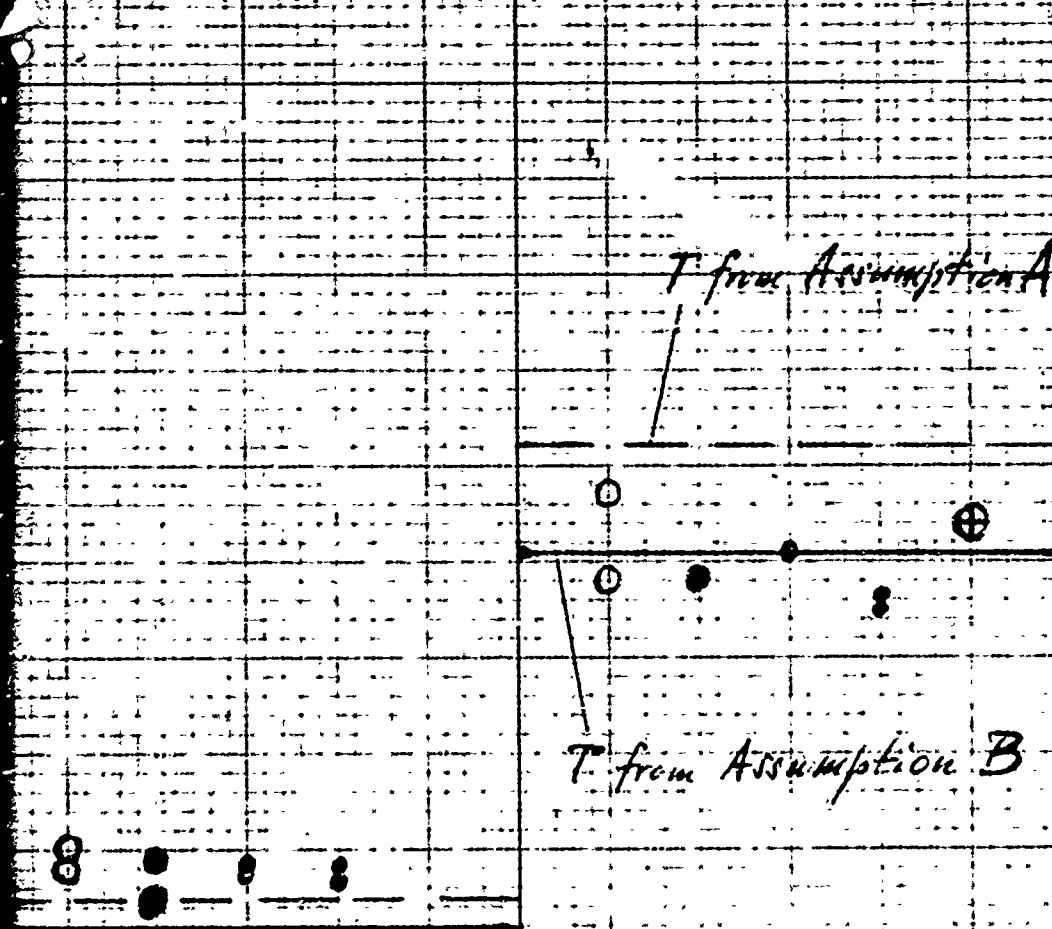
NO. 2000-10 CRYSTAL OVEN CO. MADE IN U.S.A. 10 X 10 PER INCH

- 4/5 -

$$T = \sqrt{\frac{A}{R}}$$

MM-TR-7008-F

Surge time T ,
measured vs calculated values.



M85-3

M85-7

20 mm-7

Table V : Sudden release tests:**Comparison of calculated and measured extension velocities.**

	$v_0 \propto f_2/T_1$		[inch/sec]		(28a)	
Designation/Spring	SPIW-3		SPIW-7		M85-7	
Assumption	A	B	A	B	A	B
Surge time/coil, T_1 , ms	.1585	.157	.181	.1695	.272	.246
<u>High precompression:</u>						
Measured ext.veloc. v_0	831		819		795	
Deflection/coil, f_2	.138		.131		.196	
Calculated ext.vel. v_0	871	880	724	773	721	796
Ratio meas./calcul. v_0	.954	.945	.13	1.06	1.10	1.00
<u>Medium precompression:</u>						
Measured ext.veloc. v_0	416		433		543	
Deflection/coil, f_2	.071		.071		.132	
Calculated ext.vel. v_0	448	452	392	419	485	536
Ratio meas./calcul. v_0	.929	.920	1.105	1.035	1.12	1.01
<u>Modium precompression:</u> (rear spring end fixed)						
Measured ext.veloc. v_0	437		434		565	
Deflection/coil, f_2	.071		.071		.135	
Calculated ext.vel. v_0	448	452	392	419	496	549
Ratio meas./calcul. v_0	.976	.967	1.105	1.035	1.14	1.03
Aver.ratio meas./cal. v_0	.953	.944	1.113	1.043	1.12	1.013

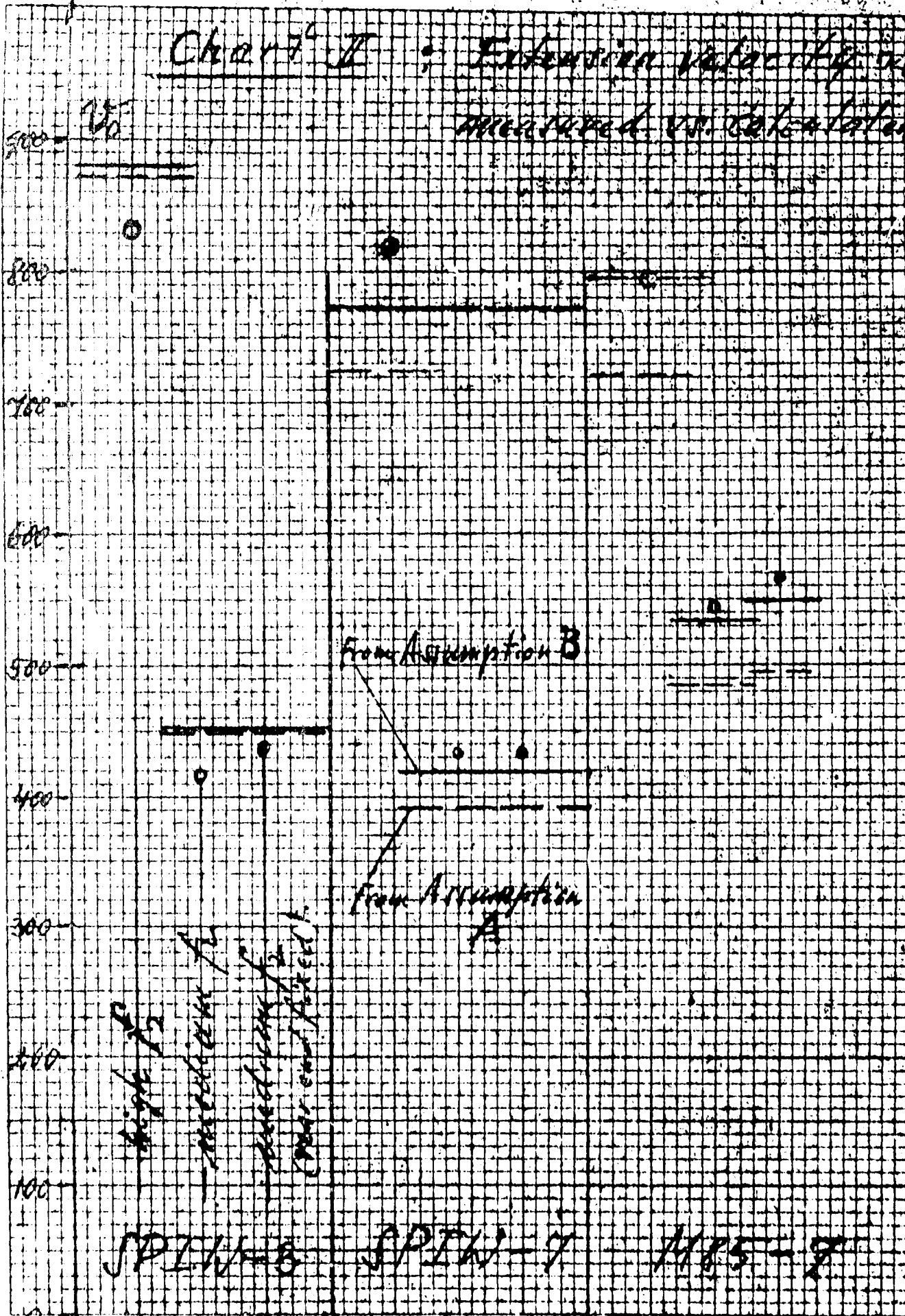
Notice difference in ratios between 3-strand and 7-strand springs.

inch/sec

-47-

Sudden release tests

Chart II : Extension velocity, v , measured vs. calculated.



- 47 -

65

MM-TR-7008-F

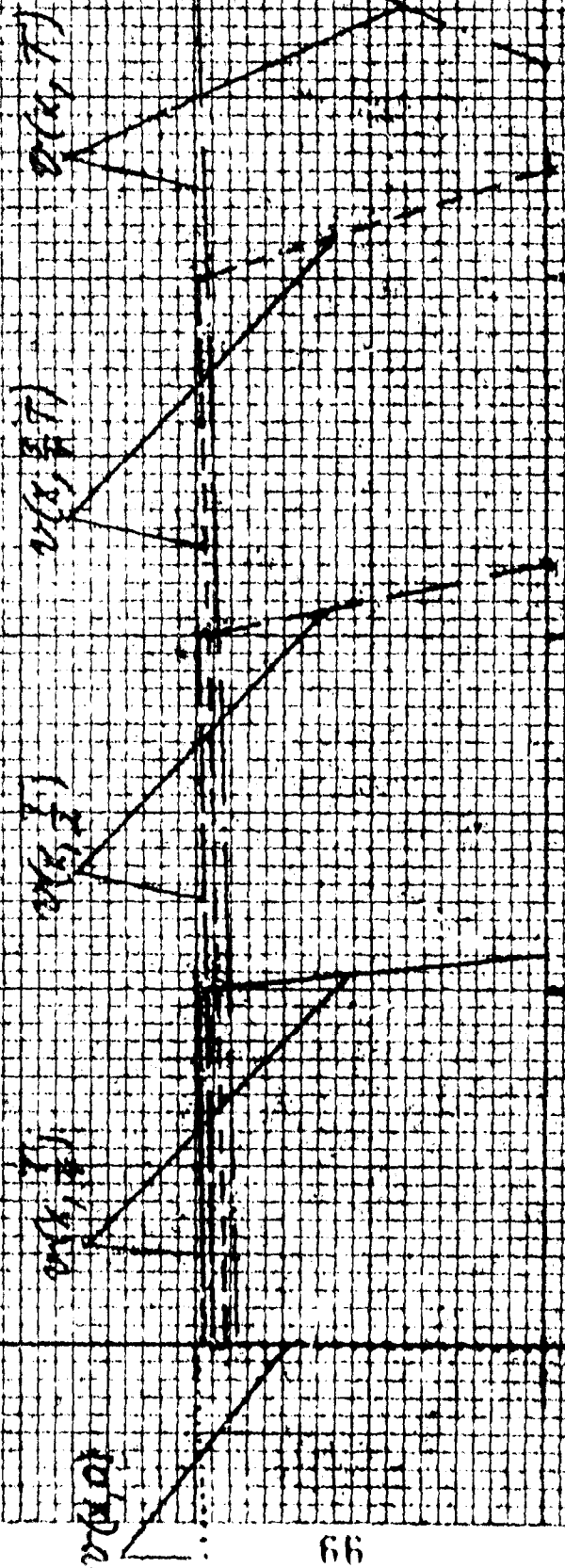
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14-TR-7004F

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Diagram of the motion of a particle in a potential well.

Velocity, or
Potential, u



At time $t = T$, wave packet has reached other boundary and wave packet is already at $x = x/4$.